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Abstract

This paper is the first to study the hedging of price risk with uncertain payment dates, a frequent problem in practice. It derives a variance-minimizing hedging strategy for two settings, the first employing linear contracts with different times to maturity and the second allowing for non-linear exotic derivatives. Using commodity prices and exchange rates, we empirically show the optimal strategy clearly outperforms heuristic alternatives in both settings. Non-linear instruments offer advantages with increasing hedge horizons and strongly dependent time and price risk, while linear instruments can suffice for short horizons and weak dependency.

JEL classification: G30; D81

Keywords: risk management; hedging; forwards; exotic derivatives; time uncertainty

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1 Introduction

There is little doubt about the benefits of a well-designed corporate risk management strategy.[1] The practice of risk management, however, can become very complicated because risk usually has different dimensions. Two examples are price risk and quantity risk, such as when the sales price or the quantity of raw materials required for production the following year are uncertain. There is, however, often a further dimension, namely, uncertainty about the timing of cash flows, such as when a firm produces for stock but does not know exactly when its products will be sold. Other examples are technological problems that can lead to uncertain development and production times for a new product line or lawsuits that could take a long time until a claim of recourse is decided.

Timing aspects have been discussed in different contexts in the finance literature, for example, the uncertainty of the default time when valuing default-risky bonds, portfolio choice problems when the investor's lifetime (time horizon) is uncertain, or the uncertain exercise time of American-style derivatives contracts. However, the literature on non-financial firms' risk management and hedging strategies has hardly considered uncertainty in the timing of cash flows.[2]

The uncertainty of cash flow timing, which we call time uncertainty or time risk in this paper, raises interesting questions about the design and performance of strategies that are supposed to hedge price risk. Since time risk (potentially) calls for dynamic strategies, what kind of adjustments are necessary over time? Under which conditions are simple static strategies adequate and under which conditions are they clearly inferior, compared to dynamic ones? Should hedging strategies use different derivative contracts with different maturities simultaneously? What are appropriate payoff profiles in the presence of time risk? Is hedging with linear contracts such as forwards and futures sufficient or should firms use options, possibly even exotic ones? How do different hedging strategies perform empirically in the presence of time uncertainty? How difficult does the hedging of price risk become in a particular market

[1] See, for example, Smith and Stulz (1985), Campbell and Kracaw (1990), Bessembinder (1991), Froot et al. (1993), DeMarzo and Duffie (1995), Ross (1996), Morellec and Smith (2007), and Stulz (2013), who provide various reasons why risk management could increase firm value. Empirical studies by, for example, Allayannis and Weston (2001), Carter et al. (2006), and Bartram et al. (2011) confirm the view that proper risk management can be beneficial for value creation.

[2] One exception is Culp and Miller's (1995) discussion on the optionality in Metallgesellschaft's long-term oil delivery contracts, which made delivery times uncertain and could have been a rationale for the firm's rollover strategy with short-term futures.

given additional time risk? These are the questions this paper addresses.

The issue of time risk is closely related to two other problems that have been discussed in the literature on corporate hedging, namely, basis risk and quantity risk. One way to look at the uncertain timing of cash flows is that it introduces a potential mismatch between the time of payment and the maturity of a hedging instrument, that is, it leads to basis risk. The effects of basis risk on hedging strategies with futures have been analyzed by, for example, Rolfo (1980), Anderson and Danthine (1980, 1981), Benninga et al. (1984), Briys et al. (1993), and Adam-Müller and Nolte (2011). Another way of looking at time uncertainty is that, at any future payment date, a promised cash flow may or may not occur, which means that the quantity to be hedged is either that promised or zero. Thus, we have to deal with a certain kind of quantity risk. Hedging problems with forward contracts under both price risk and quantity risk have been analyzed by, for example, Benninga et al. (1985), Eaker and Grant (1985), Kerkvliet and Moffett (1991), Chowdhry (1995), Adam-Müller (1997), and Brown and Toft (2002). Our analysis of time risk is based upon important results of these studies, particularly that, given uncorrelated price and quantity risk and unbiased forward markets, a variance-minimizing forward hedge is a (expected) full hedge (e.g., Benninga et al. (1985, p. 543), Eaker and Grant (1985, p. 225), and Adam-Müller (1997, p. 1424)). In contrast to the cited literature, however, hedging under time risk calls for a multi-period setting and should consider the availability of derivatives contracts of different maturities. For this reason, our theoretical analysis extends this strand of literature.

Another issue that we address in this paper is which payoff profiles of derivative contracts are optimal in the presence of time risk, that is, we ask what kind of derivative instruments should be used. The seminal work by Brown and Toft (2002) addresses this issue in a one-period setting with price risk and quantity risk and Korn (2010) extends the analysis by relaxing the distributional assumptions. Mahul (2002) and Chang and Wong (2003) study the effect of basis risk on the usage of futures and options and Moschini and Lapan (1995) investigate the hedging role of futures and options under joint price, basis, and production risk. We are the first, however, to address the issue of instrument choice in the context of time risk and derive the optimal payoff functions of derivative contracts.

Our paper also makes an empirical contribution by comparing the performance of different hedging strategies in the presence of time risk. The empirical study is based on the prices of oil, copper, and gold as well as the US dollar (USD) to euro exchange rate. It compares the optimal strategy with various alternatives, including simple heuristic strategies used in practice, under different scenarios with varying degrees of dependence between price risk and time risk. Our results show that the optimal hedging strategies clearly outperform simple heuristic alternatives. For short hedge horizons and a weak dependence between price risk and time risk, linear hedging instruments are sufficient. If the hedge horizons increase and price risk and time risk are strongly dependent, however, non-linear derivatives can lead to a significant improvement in terms of risk reduction.

The remainder of the paper is structured as follows: Section 2 introduces the model framework and derives optimal hedging policies under both price risk and time risk. Section 3 discusses the empirical results and Section 4 summarizes the main findings and presents our conclusions.

2 Optimal Hedging Policies

2.1 Setting

As a base case for the analysis of hedging policies under time risk, consider a firm with a single product. The date is currently date 0; the output quantity, Q, is already determined; and the product will be sold at a later date. However, the exact timing of the sale is uncertain. It could take place on either date 1 or date 2, with date 2 being the firm's planning horizon. The exogenous product prices[3] P_1 and P_2 at dates 1 and 2, respectively, are uncertain and the costs C are deterministic. The firm has the opportunity to finance or invest at a fixed rate r and invests all revenues that accrue prior to the planning horizon. Under these assumptions, the firm's operating profits over its planning horizon equal

$$P_1\,Q\,I\,(1+r) + P_2\,Q\,(1-I) - C, \tag{1}$$

where I is a random variable describing the uncertain timing of the sale that takes a value of one if the sale occurs at date 1 and a value of zero otherwise.

[3] The price variable could also be an exchange rate. In this case, Q represents an exporting firm's revenues in a foreign currency.

The firm has access to derivative contracts written on prices[4] that it can use to hedge the risk in its operating profits. These contracts can be entered into at dates 0 and 1 and expire either at date 1 or date 2. The terms $g_1(\cdot)$ and $g_2(\cdot)$ denote the corresponding payoffs of the derivatives in question at expiration, respectively. All derivatives are deferred payment contracts, that is, any premium is paid at maturity. Cash flows from derivatives that accrue at date 1 are either invested or financed at the rate r, leading to the following profits from the derivative contracts:

$$g_1(\cdot)(1+r) + g_2(\cdot)\,. \tag{2}$$

Combining profits from operations and profits from derivatives then yields the following total profits:

$$\begin{aligned} \Pi &= P_1\,Q\,I\,(1+r) + P_2\,Q\,(1-I) - C \\ &\quad + g_1(\cdot)(1+r) + g_2(\cdot)\,. \end{aligned} \tag{3}$$

The firm's hedging problem is to find appropriate payoff functions $g_1(\cdot)$ and $g_2(\cdot)$.

Optimal hedging policies are derived under two additional assumptions.

Assumption 1: *The expected profit from derivatives contracts is zero for all contracts and all periods.*

Under this assumption, derivatives do not offer the firm an opportunity to earn risk premiums or speculative profits based on superior information. Therefore, no speculative components will appear in the optimal hedge positions.[5] Whether this is a reasonable assumption depends on the particular market, the particular derivative, and the firm's information set. At the very least, Adam and Fernando (2006) and Brown et al. (2006) provide empirical evidence that selective hedging (speculation) by non-financial firms is unsuccessful, on average, and Bartram (2015) generally finds little evidence of speculation with derivatives for an international sample. Therefore, the assumption of zero expected profits from derivatives positions is the natural starting point to derive the pure hedging component of a firm's derivative strategy.

[4] Which prices are relevant will become apparent when we derive the optimal hedging strategies.
[5] Note that Brown and Toft (2002) make the same assumption in their analysis of optimal hedging policies.

Since speculative trading motives are ruled out by Assumption 1, the sole purpose of derivatives is risk reduction. At this point, the second assumption comes into play.

Assumption 2: *The firm minimizes the variance of total profits.*

Variance minimization is a common starting point when risk reduction needs to be made concrete.[6] Its major advantage lies in the tractability of the corresponding optimization problem, which often leads to closed-form solutions that are easy to interpret and implement.

2.2 Optimal Linear Hedge

In a first step, we concentrate on linear hedging instruments, such as forwards or futures, which are easy to understand and often traded in highly liquid markets. Therefore, many non-financial firms primarily use linear hedging instruments for their risk management. Gay et al. (2002) report that 69% of non-financial firms in their sample use only linear derivatives to manage commodity price risk and 75% of firms rely solely on linear derivatives to hedge exchange rate risk. Similarly, Huang et al. (2007) find that 73% of a group of firms that use derivatives to manage either currency or interest rate risks use linear instruments exclusively. Bartram et al. (2011) analyze a hand-collected sample of 6,888 firms from 47 countries and find that forwards and futures are the preferred instrument for hedging foreign exchange (FX) risk and commodity price risk, respectively. This holds for both the overall sample and US sub-sample.

To solve the hedging problem, we have to consider hedging policies at dates 0 and 1. At date 1, we assume that the firm has access to forward contracts on price P_2, that is, the payoff function $g_2(\cdot)$ becomes

$$g_2(\cdot) = (P_2 - F_{1,2})\, h_{1,2}, \tag{4}$$

[6] Of course, variance minimization can be criticized on theoretical grounds. One aspect is that variance minimization is compatible with expected utility maximization only under certain assumptions about utility functions and price distributions. Moreover, variance minimization could lead to time inconsistencies in multi-period problems (for a discussion of the latter issue, see, e.g., Basak and Chabakauri (2010)). Under our assumptions, however, the resulting hedging strategy is time consistent.

where $F_{1,2}$ is the forward price at date 1 and $h_{1,2}$ denotes the number of contracts bought. Note that $g_2(\cdot)$ also indicates the profit of the contract between dates 1 and 2.

At date 0, the firm has access to two different forward contracts written on P_1 and P_2, respectively, with maturity dates 1 and 2. The profit of these contracts over the period from date 0 to date 1 equals

$$g_1(\cdot) = (P_1 - F_{0,1})\, h_{0,1} + (F_{1,2} - F_{0,2})\,(1+r)^{-1}\, h_{0,2}, \tag{5}$$

where $F_{0,1}$ and $F_{0,2}$ denote the forward prices and $h_{0,1}$ and $h_{0,2}$ the number of contracts bought.[7] With these specifications, the firm's total profits over the planning horizon become

$$\begin{aligned} \Pi \;=\; & P_1\, Q\, I\, (1+r) + P_2\, Q\,(1-I) - C \\ & +(P_1 - F_{0,1})(1+r)\, h_{0,1} + (F_{1,2} - F_{0,2})\, h_{0,2} + (P_2 - F_{1,2})\, h_{1,2}. \end{aligned} \tag{6}$$

Given the total profits from Equation (6), the firm has to solve the optimization problem

$$\min_{h_{0,1},\; h_{0,2},\; h_{1,2}} Var_0[\,\Pi\,]. \tag{7}$$

Variance minimization according to Equation (7) poses a dynamic optimization problem that has to be solved recursively, using the information available on the two decision dates. This problem is solved in the Appendix. The following proposition provides the solution.

Proposition 1: *The optimal hedging policy with linear contracts is given by*

$$h^*_{1,2} = -Q\,(1-I)\,, \tag{8}$$

[7] Note that the profit of the two-period contract over the first period equals the discounted forward price change.

$$
\begin{pmatrix} h^*_{0,1} \\ h^*_{0,2} \end{pmatrix} = -Q \Bigg[\begin{pmatrix} q \\ 1-q \end{pmatrix}
+ \begin{pmatrix} Var_P & Cov_{P\,F} \\ Cov_{P\,F} & Var_F \end{pmatrix}^{-1} \begin{pmatrix} Cov_{P\,I} \cdot (F_{0,1}(1+r) - F_{0,2}) \\ Cov_{F\,I} \cdot (F_{0,1}(1+r) - F_{0,2}) \end{pmatrix}
+ \begin{pmatrix} Var_P & Cov_{P\,F} \\ Cov_{P\,F} & Var_F \end{pmatrix}^{-1} \begin{pmatrix} CoSk_{P\,P\,I} - CoSk_{P\,F\,I} \\ CoSk_{P\,F\,I} - CoSk_{F\,F\,I} \end{pmatrix} \Bigg], \tag{9}
$$

where q is the unconditional probability that the sale occurs on date 1; Var_P, Var_F, and $Cov_{P\,F}$ are the variances and covariance of $P_1\,(1+r)$ and $F_{1,\,2}$; and $Cov_{P\,I}$ and $Cov_{F\,I}$ denote the covariances between the time risk indicator I and the two price variables $P_1\,(1+r)$ and $F_{1,\,2}$, respectively. Finally, the terms $CoSk_{P\,P\,I}$, $CoSk_{P\,F\,I}$, and $CoSk_{F\,F\,I}$ are measures of coskewness defined as $CoSk_{P\,P\,I} := E[(P_1(1+r) - F_{0,1}(1+r))^2(I-q)]$, $CoSk_{P\,F\,I} := E[(P_1(1+r) - F_{0,1}(1+r))(F_{1,2} - F_{0,2})(I-q)]$, and $CoSk_{F\,F\,I} := E[(F_{1,2} - F_{0,2})^2(I-q)]$.

Proposition 1 provides insight into the structure of optimal forward hedges under both price risk and uncertainty about the payment date. (i) The quantity Q is just a linear scaling factor of the hedge positions. (ii) The hedge position $h^*_{1,2}$ at date 1 guarantees that no risk remains in the second period. If the sale has already been made at date 1, there is no price risk and $h^*_{1,2} = 0$. If the sale will occur at date 2, the remaining price risk will be fully hedged and $h^*_{1,2} = -Q$. (iii) The hedging strategy followed at date 0 is more complicated. At date 0, time risk has not been dissolved and the strategy has to consider both price risk and time risk. Generally, the optimal strategy uses both forward contracts simultaneously, that is, positions in two derivatives contracts with different times to maturity are taken to hedge the risk inherent in one payment. This characteristic of the optimal strategy clearly distinguishes it from heuristic alternatives that are used in practice to deal with time uncertainty. (iv) The optimal strategy requires an adjustment of positions at date 1 and is therefore a truly dynamic strategy. (v) The optimal hedge position at date 0 comprises three components. The first component consists of the unconditional probabilities of a sale at date 1 (q) and date 2 ($1-q$) multiplied by $-Q$. This result is intuitively appealing, because a high probability of a sale at date 1 makes the short-term forward the natural instrument to use. To the contrary, if the probability of a sale at date 2 is high, the long-term contract is the natural hedging instrument. (vi) In addition, the firm could try to hedge some of the time risk with forward contracts. Such an attempt shows up in the second component. It can be interpreted as the vector of regression coefficients from a multiple linear regression of $I(F_{0,1}(1+r) - F_{0,2})$ on P_1 and $F_{1,2}$. Note that $F_{0,1}(1+r) - F_{0,2}$ is the date 0 expectation of $P_1(1+r) - F_{1,2}$, which is the firm's remaining exposure to time risk, given the optimal hedging strategy in the second period. (vii) The third component involving the coskewness terms arises because the link between price risk and time risk is multiplicative. (viii) In summary, setting up the optimal linear hedging strategy at date 0 can be split into the following three steps: (1) Use the expected exposure to price risk in the first period and hedge it fully, (2) use the expected exposure to time risk in the first period and hedge it as much as possible with linear instruments written on prices, and (3) account appropriately for the fact that there is a multiplicative link between price risk and time risk.

As a direct implication of Proposition 1, Corollary 1 shows that a substantial reduction in the complexity of the optimal hedging strategy results if uncertainty about the payment date is independent of price risk. Such a case is realistic under certain circumstances, such as when time risk results from a project's technological problems.

Corollary 1: *If price risk and time risk are independent, the optimal hedging policy with linear contracts is given by*

$$h^*_{1,2} = -Q\,(1 - I)\ , \quad h^*_{0,1} = -Q\,q\ , \quad h^*_{0,2} = -Q\,(1-q)\ . \tag{10}$$

Compared to the general case with dependent time risk and price risk, the hedge positions at date 0 do not depend on any characteristic of the joint distribution of P_1 and $F_{1,2}$. Moreover, both forward positions are generally sell positions and the firm sells exactly Q contracts in total. In this sense, the firm's output is fully hedged.

Although the optimal hedging strategy according to Corollary 1 is relatively simple, it requires the firm's estimate of the probability of a sale at date 1. This is a disadvantage compared to alternative heuristic strategies. A first important alternative is a rollover strategy that takes a position of $-Q$ contracts in the one-period forward at date 0. If the sale occurs at date 1, the strategy is terminated. If the sale does not occur at date 1, the forward position is rolled over in a new one-period forward contract. A second important alternative is to sell $-Q$ contracts in a long-term (two-period) forward and to close out the position as soon as the sale occurs.

Fortunately, the optimal strategy shows some robustness with respect to imprecise estimates of the probability q. Presume that the firm uses the estimate $\hat{q}$ instead of the true parameter q. Then the optimal strategy implemented with $\hat{q}$ still leads to a lower variance than the rollover strategy if $\hat{q} > 2q - 1$ and to a lower variance than the long-term strategy if $\hat{q} < 2q$. This result is proven in the Appendix and illustrated in Figure 1.

[Insert Figure 1 about here]

As Figure 1 shows, there are large regions where the optimal strategy conveys advantages in terms of variance reduction (light regions) compared to a rollover strategy or a long-term strategy, even if $\hat{q}$ has estimation errors. Only if the true probability of a sale at date 1 is high and the estimated probability rather low, does the rollover strategy perform better. For example, if the true probability equals 0.75 and the estimated one is 0.5, the optimal strategy still achieves the same variance as the rollover strategy. The good performance of the rollover strategy for high values of q is natural, because the rollover strategy becomes optimal if $q = 1$. The long-term strategy is the exact mirror image of the rollover strategy. It performs well for low values of q and becomes the optimal strategy if $q = 0$.

2.3 Optimal Exotic Hedge

The second step of our analysis lifts the restriction to linear hedging instruments and allows the firm to enter into derivatives with any linear or non-linear payoff structure (exotic hedges). At date 1, derivative contracts written on P_2 are available, the only risk remaining at that time. At date 0, the firm can enter into contracts on any price risks relevant in the first period, namely, P_1 and $F_{1,2}$.[8] Under these assumptions, the firm's optimization problem becomes

$$\min_{g_1(P_1, F_{1,2}),\, g_2(P_2)} Var_0[\Pi], \quad s.t. \quad E_0[g_1(P_1, F_{1,2})] = E_1[g_2(P_2)] = 0. \tag{11}$$

Note that problem (11) is a dynamic functional optimization problem, which requires finding whole payoff functions instead of single parameters. The following proposition states the solution to the problem. A proof is provided in the Appendix.

8 In Section 2.4, we discuss the effect of having available even more derivatives contracts written on other price variables.

Proposition 2: *The optimal exotic hedge is given by*

$$g_2^*(P_2) \;=\; -Q\,(1-I)\,(P_2 - F_{1,2})\,, \tag{12}$$

$$g_1^*(P_1, F_{1,2}) \;=\; -Q\,\left[q_{\,|\,P_1\,F_{1,2}}\,P_1 \,+\, (1-q_{\,|\,P_1,\,F_{1,2}})\,F_{1,2}\,(1+r)^{-1}\right] - k\,, \tag{13}$$

where $q_{\,|\,P_1,\,F_{1,2}}$ *denotes the conditional probability (given* P_1 *and* $F_{1,2}$*) of a sale at date* 1 *and* k *is a constant parameter ensuring that the expected profit of the derivatives contract is zero.*

Some properties of the optimal exotic hedge are worth mentioning: (i) The optimal derivative position in the second period is still the same linear hedge as in Proposition 1. The reason is that such a hedge already eliminates risk completely in the second period and no improvement is possible. (ii) The crucial element of the hedge in the first period is the conditional probability $q_{\,|\,P_1\,F_{1,2}}$ of a payment at date 1. This conditional probability needs to be specified by the firm. (iii) The optimal hedge will generally deviate from the optimal linear hedge of Proposition 1, because the conditional probability $q_{\,|\,P_1\,F_{1,2}}$ can be a complex non-linear function of the prices. Therefore, time risk provides an explanation for the use of non-linear derivatives.[9] (iv) If price risk and time risk are independent, however, the conditional probability equals the unconditional probability q and we obtain a substantial reduction in complexity, as stated in Corollary 2.

Corollary 2: *If price risk and time risk are independent, the optimal exotic hedge is the linear hedge, as given in Corollary 1.*

Corollary 2 stresses the importance of the rather simple linear hedging strategy that combines positions in a short-term forward and a long-term forward according to the probability of an early sale. If price and time risk are independent, such a strategy is not only the optimal linear hedging policy but also the best one generally accessible with derivatives. Therefore, if firms do not consider dependencies between prices and the timing of cash flows in their hedging decisions, this result supports the major role of linear instruments in risk management.

[9] Of course, there are several reasons to use non-linear derivatives, despite dependence between price risk and time risk, such as quantity risk (Brown and Toft (2002), Korn (2010)), specific forms of basis risk (Mahul (2002), Chang and Wong (2003), Adam-Müller and Nolte (2011)), credit risk of derivatives (Mahul and Cummins (2008)), liquidity risk arising from marking to market (Adam-Müller and Panaretou (2009)), a firm's general financial constraints (Froot et al. (1993), Adam (2002), Adam (2009)), and regret aversion (Korn and Rieger (2016)).

The following example illustrates optimal exotic hedges for dependent price and time risk. We parameterize the conditional probability of a payment at date 1 as a logistic function of the product's relative price change in the first period,

$$q_{|P} = \frac{\exp(-\beta(\frac{P_1 - P_0}{P_0}))}{1 + \exp(-\beta(\frac{P_1 - P_0}{P_0}))}, \tag{14}$$

where β is a parameter governing the dependence between price risk and time risk. The rationale behind Equation (14) is as follows: The potential buyer of the product conditions the decision to buy early (date 1) upon assessment of the price being rather high or rather low at date 1. If the price has increased in the previous period, it will be judged as relatively high and the purchase will have a higher probability of being deferred. If the price has decreased, it will be seen as currently favorable and an immediate purchase will have a higher probability.

We consider three scenarios with different parameters values for β, using an initial price $P_0 = 1$. For $\beta = 0$, price risk and time risk are independent and the probability of a sale at date 1 equals 0.5. A β value of four represents moderate dependence. In this case, if the price has risen by 10% in the previous period, the probability of an immediate sale will only be 0.4. With $\beta = 8$, we have strong dependence, meaning that a price increase of 10% results in a conditional probability of about 0.2. Figure 2 shows the optimal payoff functions for the three scenarios.

[Insert Figure 2 about here]

The upper part of Figure 2 refers to the case of independent price and time risk. It confirms that the optimal hedge has payoffs that are linear in both price variables. The middle and lower parts of the figure illustrate the effects of dependence. Since the conditional probability in Equation (14) does not depend on $F_{1,2}$, the optimal payoff function is linear in $F_{1,2}$ for every fixed value of P_1. However, we observe strong non-linearities in P_1. The optimal payoff function is increasing with P_1 in some regions and decreasing in others. Moreover, it has both concave and convex areas. The specific form of the payoff function can be better understood if we consider an extremely high beta. In this case, we can be confident that a value of P_1 below one almost surely leads to an immediate sale, making a full hedge with short-term forwards the best way to proceed. Conversely, for a value of P_1 above one, the sale will almost surely take place at date 2 and a full hedge with long-term forwards is optimal. This is the reason for the "kink" at $P_1 = 1$ in the lower part of Figure 2.

2.4 Multiple Products

Because most firms have more than one product, whether our conclusions about optimal hedging policies change if we consider multiple products and therefore allow for multiple price and time risks is an important question.

For ease of notation, we concentrate on two products, A and B; however, the results are easily generalizable. Variables with a subscript or superscript A refer to the first product and those with a subscript or superscript B refer to the second. We assume that derivatives written on prices exist for both products. The following equation shows the firm's total profits in the two-product case, where all the random variables $P_2^A, P_2^B, P_1^A, P_1^B, F_{1,2}^A, F_{1,2}^B, I_A$, and I_B can be stochastically dependent:

$$\begin{aligned} \Pi \;=\;& P_1^A\, Q_A\, I_A\,(1+r) + P_2^A\, Q_A\,(1-I_A) - C_A \\ & + P_1^B\, Q_B\, I_B\,(1+r) + P_2^B\, Q_B\,(1-I_B) - C_B \\ & + g_1(P_1^A, F_{1,2}^A, P_1^B, F_{1,2}^B)(1+r) + g_2(P_2^A, P_2^B)\,. \end{aligned} \tag{15}$$

The firm's goal is to find the hedging policy that minimizes the variance of its profits in Equation (15). Proposition 3 provides solutions for both the optimal linear hedge and the optimal exotic hedge. The proof is entirely analogous to the proofs of Propositions 1 and 2.

Proposition 3: (i) *The optimal hedging policy with linear contracts is given by*

$$h_{1,2}^{A*} = -Q_A\,(1-I_A)\;,\; h_{1,2}^{B*} = -Q_B\,(1-I_B)\;, \tag{16}$$

$$\begin{pmatrix} h_{0,1}^{A*} \\ h_{0,2}^{A*} \\ h_{0,1}^{B*} \\ h_{0,2}^{B*} \end{pmatrix} = -\begin{pmatrix} Q_A \\ Q_A \\ Q_B \\ Q_B \end{pmatrix} \left[\begin{pmatrix} q_A \\ 1-q_A \\ q_B \\ 1-q_B \end{pmatrix} \right. \tag{17}$$

$$
+\begin{pmatrix} Var_{P_A} & Cov_{P_AF_A} & Cov_{P_AP_B} & Cov_{P_AF_B} \\ Cov_{P_AF_A} & Var_{F_A} & Cov_{F_AP_B} & Cov_{F_AF_B} \\ Cov_{P_AP_B} & Cov_{F_AP_B} & Var_{P_B} & Cov_{P_BF_B} \\ Cov_{P_AF_B} & Cov_{F_AF_B} & Cov_{P_BF_B} & Var_{F_B} \end{pmatrix}^{-1} \left[\left[\begin{pmatrix} Cov_{P_AI_A} & Cov_{P_AI_B} \\ Cov_{F_AI_A} & Cov_{F_AI_B} \\ Cov_{P_BI_A} & Cov_{P_BI_B} \\ Cov_{F_BI_A} & Cov_{F_BI_B} \end{pmatrix} \begin{pmatrix} F^A_{0,1}(1+r) - F^A_{0,2} \\ F^B_{0,1}(1+r) - F^B_{0,2} \end{pmatrix}\right.\right.
$$

$$
\left.\left. + \begin{pmatrix} CoSk_{P_AP_AI_A} - CoSk_{P_AF_AI_A} + CoSk_{P_AP_BI_B} - CoSk_{P_AF_BI_B} \\ CoSk_{F_AP_AI_A} - CoSk_{F_AF_AI_A} + CoSk_{F_AP_BI_B} - CoSk_{F_AF_BI_B} \\ CoSk_{P_BP_AI_A} - CoSk_{P_BF_AI_A} + CoSk_{P_BP_BI_B} - CoSk_{P_BF_BI_B} \\ CoSk_{F_BP_AI_A} - CoSk_{F_BF_AI_A} + CoSk_{F_BP_BI_B} - CoSk_{F_BF_BI_B} \end{pmatrix}\right]\right],
$$

where q_A and q_B are the unconditional probabilities for the sales of products A and B at date 1, respectively. The variances, covariances, and coskewnesses are defined in the same way as the corresponding terms in Proposition 1.

(ii) *The optimal exotic hedge is given by*

$$
g_2^*(P_2^A, P_2^B) \;=\; -Q_A\,(1-I_A)\,(P_2^A - F_{1,2}^A) - Q_B\,(1-I_B)\,(P_2^B - F_{1,2}^B)\;, \quad (18)
$$

$$
\begin{aligned}
& g_1^*(P_1^A, F_{1,2}^A, P_1^B, F_{1,2}^B) = & (19)\\
& \quad -Q_A\left[q_{A\,|\,P_1^A,F_{1,2}^A,P_1^B,F_{1,2}^B}\,P_1^A \;+\; (1-q_{A|\,P_1^A,F_{1,2}^A,P_1^B,F_{1,2}^B})\,F_{1,2}^A\,(1+r)^{-1}\right] \\
& \quad -Q_B\left[q_{B\,|\,P_1^A,F_{1,2}^A,P_1^B,F_{1,2}^B}\,P_1^B \;+\; (1-q_{B|\,P_1^A,F_{1,2}^A,P_1^B,F_{1,2}^B})\,F_{1,2}^B\,(1+r)^{-1}\right] - k,
\end{aligned}
$$

where $q_{A\,|\,P_1^A,F_{1,2}^A,P_1^B,F_{1,2}^B}$ denotes the conditional probability (given all prices) of a sale of product A at date 1 and $q_{B\,|\,P_1^A,F_{1,2}^A,P_1^B,F_{1,2}^B}$ denotes the corresponding probability for product B.

Proposition 3 shows that the general structure of the optimal hedges is the same as in the one-product case. The linear hedge still consists of three components and the conditional probability of a sale at date 1 is still the central element of the optimal exotic hedge. A very important property of optimal hedging policies for both the linear and the exotic case is that any dependence between the time risks of the two products is irrelevant. Consequently, the firm can treat the two products separately and set up independent hedging policies for each of them, a considerable reduction in complexity. Such a separation of hedging policies for the two products can be seen directly if price risks and time risks are independent. In this case, we obtain the forward hedges of Corollary 1 as the optimal policy for each product. In the case of dependent price and time risks, the separation still exists, although the optimal policies are different from those shown in Propositions 1 and 2. Since this statement could seem contradictory at first sight, one must bear in mind that we not only added another product but also allowed for a larger set of derivatives written on a larger set of prices. If a derivative written on P_1^B is useful to cross-hedge the time risk of product A, it will be used in the optimal hedging policy, even if the firm is a single-product firm and does not produce B at all. Therefore, proof of separation is eventually provided by the comparison of the two-product firm with two single-product firms that have access to the same set of derivatives as the two-product firm. The resulting hedging strategies are indeed identical.

In summary, the analysis of multiple products conveys two central messages: (i) A multi-product firm can devise hedging policies for all products separately and will still attain the overall optimal strategy and (ii) the firm should consider all available derivatives that help to cross-hedge time risk, not only those written on the product price itself.

3 Hedging Performance

An important question of this paper is how the derived optimal hedging strategies perform empirically. It has been shown that the strategy set forth in Corollary 1 will have the lowest variance if price and time risk are independent, and that the strategy formulated in Proposition 2 will minimize variance in the case of time–price dependence. It has not yet been determined, however, by how much these policies will outperform alternatives in a real world context. We therefore investigate several aspects of hedging performance: In a first step, we look at the case of independent price and time risk. Here we compare the hedging effectiveness of the optimal strategy over different markets and examine by how much it improves on alternative heuristic hedging strategies that are used in practice. In a second step, we introduce dependence between time and price risk and analyze customized non-linear contracts. Again, we compare hedging performance to some heuristics, including the optimal linear strategy of Proposition 1.

3.1 Data and Design

To investigate the empirical performance of hedging strategies, we must specify the firm's planning horizon, price risk, and the timing uncertainty of revenues. For our base case, we assume that the firm's planning horizon is one year, which seems appropriate, since empirical evidence shows that many firms predominantly use derivatives with maturities of one year or less.[10] This evidence could indicate that longer-term exposure is managed differently, for example, by means of operational hedging. It is further assumed that the planning horizon consists of two periods of equal length and revenues can occur at the end of each; that is, in the base case, they occur either after six months or after 12 months.

No particular distributional assumption is made to capture price risk. Instead, our study uses 25 years of historical price data from several commodity markets and the FX market. In particular, we use

- West Texas Intermediate crude oil futures
- Copper futures
- Gold futures
- USD–euro spot and forward exchange rates.

Oil, copper, and gold are selected because the role of convenience yield and convenience yield risk is quite different for the three commodities. As Schwartz (1997) and Casassus and Collin-Dufresne (2005) show, convenience yields are most important for oil and least important for gold. A time-varying convenience yield constitutes an important risk factor for commodity derivatives in addition to the spot price and could strongly affect hedging performance. For FX derivatives, the differential between interest rates in the two currency regions is the crucial risk factor beyond the spot exchange rate.

For all commodities, we use monthly futures prices[11] of contracts with appropriate maturities from January 1990 to January 2015. The oil futures we use trade on the New York Mercantile Exchange (NYMEX) and the copper and gold futures on the New York Commodities Exchange (COMEX). Price data were supplied directly by the Chicago Mercantile Exchange (CME) group.

The selected dates within a month refer to the last trading day of the expiring futures contract. Therefore, the price of the expiring contract is a sensible proxy for

[10] Compare the survey results of Bodnar et al. (1998) and Bodnar and Gebhardt (1999) for non-financial firms in the United States and Germany. Adam et al. (2016) show that, in their sample of gold mining firms, hedging activity decreases sharply with the hedge horizon and mainly concentrates on next year's production.

[11] Note that futures prices are treated as forward prices in this study. See, for example, Cox et al. (1981) for conditions under which this assumption is reasonable.

the spot price. The spot and forward exchange rates of the USD versus the euro[12] were obtained from Thomson Reuters Datastream and also cover the period between January 1990 and January 2015.

Time risk is integrated in two different ways: In the first part of the study, time and price risk are assumed to be independent and only linear contracts are considered. Here, time risk is captured by q, the probability of a sale taking place at the end of the first period, which we set equal to $0.1, 0.2, \ldots, 0.9$ to allow for a wide range of values. In the second part, dependence has to be modeled, which will be done in the same way as illustrated in Section 2.3. We use the function from Equation (14) to describe the conditional probability of a sale at date 1. By setting the parameter β equal to four and eight, we generate two scenarios, with moderate and strong dependence, respectively.

This study considers six different hedging strategies. *1) Optimal hedge:* This is the variance-minimizing strategy established in Proposition 2. *2) Optimal linear hedge:* This is the linear variance-minimizing strategy established in Proposition 1. If price risk and time risk are independent, this strategy is identical to strategy 1. *3) Unconditional linear hedge:* This hedging strategy uses the unconditional probability q to determine the positions held in forward contracts. If price risk and time risk are independent, this strategy is identical to strategies 1 and 2. *4) Rollover hedge:* Under this strategy, a full hedge in the six-month forward is placed at date 0. If the sale occurs at the end of the first period, no new forward is entered into at $t = 1$. If the sale does not occur at date 1, the forward position is rolled over into the next six-month contract. *5) Long-term hedge:* With such a strategy, the firm adopts a full hedge position in a 12-month forward at date 0 and, thus, the maturity date of the contract coincides with the last possible date of sale for the product. The forward position remains unchanged until date 2 if no sale occurs at date 1. If the product is sold at date 1, the hedge position is closed out by taking an offsetting long forward position. *6) Deferred hedge:* Under this strategy, hedging will be deferred until the exposure is known exactly. In other words, no forward contract will be entered into at $t = 0$. If the sale occurs at the end of the first period, no forward will be needed for the second period either. Only if the sale has not taken place will a full forward hedge be entered into for the second period. As a reference point, we also consider that the firm does not hedge at all. The risk of an unhedged position will be used to assess the hedging effectiveness of the optimal strategy for different markets.

[12] Prior to the introduction of the euro, exchange rates based on the Deutschmark were used.

Given our assumptions, we can derive the profit variances that result from the different hedging strategies and the no-hedge strategy. These variances are provided in the following equations, derived in the Appendix:

$$\begin{aligned}
Var_{opt} &= E_0\Big[(P_1(1+r)-F_{1,2})^2\, q_{|P}\,(1-q_{|P})\Big], \qquad (20)\\
Var_{opt\ lin} &= Var_{opt} + Var_0\Big[(q_{|P}+h^*_{0,1})\,P_1(1+r)+(1+h^*_{0,2}-q_{|P})\,F_{1,2}\Big],\\
Var_{uncon\ lin} &= Var_{opt} + Var_0\Big[(q_{|P}-q)\,(P_1(1+r)-F_{1,2})\Big],\\
Var_{roll\,over} &= Var_{opt} + Var_0\Big[(q_{|P}-1)\,(P_1(1+r)-F_{1,2})\Big],\\
Var_{long\,term} &= Var_{opt} + Var_0\Big[q_{|P}\,(P_1\,(1+r)-F_{1,2})\Big],\\
Var_{deferred} &= Var_{opt} + Var_0\Big[q_{|P}\,P_1(1+r)+(1-q_{|P})\,F_{1,2}\Big],\\
Var_{no\ hedge} &= E_0\Big[(P_1(1+r)-P_2)^2\, q_{|P}\,(1-q_{|P})\Big]\\
&\quad + Var_0\Big[q_{|P}\,P_1(1+r)+(1-q_{|P})\,P_2\Big],
\end{aligned}$$

where q is the unconditional probability of a sale at date 1 and $q_{|P} := E_0[I \mid P_1, F_{1,2}]$ is the conditional probability, that is, a random variable depending on P_1 and $F_{1,2}$. In the case of independent time and price risk, $q_{|P} = q$. The terms $h^*_{0,1}$ and $h^*_{0,2}$ are the optimal hedge positions from Proposition 1. The firm's borrowing and lending rate r is set equal to 5% p.a.

For each market and each month in the data period, we calculate percentage returns of spot and futures contracts. Starting from prices observed at the end of our data period (January 2015), we use these returns to create scenarios for the development of spot and futures prices. Because six-month futures do not trade every month for gold and some values are missing for oil and copper, the number of scenarios varies between the markets. We have 291 scenarios for oil, 293 for copper, 147 for gold, and 295 for the exchange rate. From these price scenarios, the above variances are finally calculated based on the corresponding sample moments. Note that the variance of the optimal linear hedge is slightly more difficult to obtain than the variances of the other strategies, because $h^*_{0,1}$ and $h^*_{0,2}$ depend on the covariances and coskewness measures between I and the price variables $P_1(1+r)$ and $F_{1,2}$. These covariances and coskewness measures are calculated in a first step by simulating the realization of I using the appropriate $q_{|P}$s and then bootstrapping matched pairs of these realizations and the price variables.

3.2 Results: Independent Time and Price Risk

3.2.1 Hedging Effectiveness

To measure the risk reduction achieved by the optimal hedging strategy, we use a slightly altered version of what is known as the Johnson measure:[13]

$$1 - \frac{\text{standard deviation of hedged position}}{\text{standard deviation of unhedged position}}. \tag{21}$$

This simple measure of hedging effectiveness, which fits well with the goal of variance minimization, calculates the volatility reduction of profits achieved by the optimal strategy relative to an unhedged position.

[Insert Table 1 about here]

The results presented in Table 1 confirm that the optimal strategy can indeed achieve very high levels of hedging effectiveness for all markets, regardless of the value of q. The values range from 82.62% for oil and $q = 0.6$ up to 97.53% for the exchange rate and $q = 0.6$, indicating that, for all markets and all specifications of time risk, substantial reductions in standard deviation can be attained. Looking more closely at the different commodities, it can be seen that the effectiveness is highest for gold and then copper and lowest for oil. This finding is consistent with the importance of convenience yield risk in hedging. Hedging effectiveness is lowest for the commodity with the highest convenience yield risk (oil) and highest for the commodity with the lowest convenience yield risk (gold). Moreover, the effectiveness of the foreign currency hedge, where convenience yields do not affect forward prices, is consistently higher than for the commodities.

[Insert Table 2 about here]

[Insert Table 3 about here]

As a robustness check, the firm's planning horizon has been changed from one year to two months and two years, respectively. The results are presented in Tables

[13] The original measure use variance instead of standard deviation (see Johnson (1960)).

2 and 3. In the former case, hedging effectiveness is higher across the board for the commodities with the ordering of the markets in terms of hedging effectiveness remaining unchanged. For a planning horizon of two years, the hedging effectiveness is lower than in the base case, but the ordering of the markets again stays the same. For the exchange rate, both shorter and longer horizons reduce hedging effectiveness, particularly for the two-year horizon. This result can be explained as follows: Hedging effectiveness is negatively affected by the variation in the interest rate differential between the United States and the euro zone. If the hedge horizon lengthens (shortens), there are two effects: First, the longer (shorter) the time between date 0 and date 1, the more likely are larger (smaller) interest rate changes within this period. Second, if the hedge horizon shortens, the relevant interest rates are the one-month rates instead of the six-month rates and the former show stronger variation over time. For a two-month planning horizon, this second effect seems to overcompensate for the first effect, whereas, for the two-year planning horizon, both effects should lead to lower hedging effectiveness.

3.2.2 Improvements on Alternative Strategies

As shown in the previous section, the optimal strategy allows for high levels of hedging effectiveness; however, whether it really reduces risk compared to simple heuristic alternatives has yet to be evaluated. Table 4 presents the percentage reduction in standard deviations if one switches from either the long-term, the rollover, or the deferred strategy to the optimal strategy. Several conclusions can be drawn from this table: First, risk can often be reduced substantially by adopting the optimal strategy for all markets and all values of q. Second, the highest levels of risk reduction can be achieved when replacing the deferred strategy with the optimal strategy. This finding is also consistent over all qs and all markets. The third result is that the values for long-term and rollover strategies are exact opposites with respect to the value of q. Assuming the probability of a sale after the first period is 10%, it is clear that a rollover hedge is less well suited to reduce exposure and therefore a change to the optimal strategy will yield great improvements. Should the likelihood of a sale at $t = 1$ be 90%, however, a long-term strategy is less appropriate and, consequently, the potential for risk reduction via the optimal strategy is higher.

[Insert Table 4 about here]

In many instances, there will be no telling when a sale is more likely to occur. Therefore, $q = 0.5$ represents a good starting point for the evaluation. Even in this scenario of highest possible uncertainty, the standard deviation of the hedge position can be reduced by about 6% for the USD–euro exchange rate and by as much as 25% for oil when the current strategy is a long-term or a rollover hedge. If the deferred hedge has been the strategy of choice, reductions of up to 92% for gold are made possible by the optimal hedge. The ordering of the markets is not as clear as with the hedging effectiveness but depends on the alternative strategy. After crude oil, copper exhibits the most potential for volatility reduction for the long-term and rollover strategies, whereas a greater reduction can be achieved for gold when switching from the deferred strategy.

With probabilities other than 0.5, even greater reductions are possible for firms following a deferred strategy and, depending on whether the chance of a sale at $t = 1$ increases or decreases, for the other two strategies as well. When hedging exposure to gold price risk with the likelihood of a sale at $t = 1$ being 0.9, for example, the standard deviation can be reduced by 95% for the deferred hedge and by 50% for the long-term hedge. For the deferred hedge, the values are only slightly lower for the other markets and, with respect to the long-term strategy, volatility reductions of up to 65% are possible for crude oil, for example.

Again, we change the planning horizon for robustness checks. In a setup with two one-month periods, as reported in Table 5, similar improvements can be achieved by opting for the optimal strategy. In the case of the deferred hedge, volatility can be reduced even more than in the previous setting. For oil and copper, this also holds for the long-term and the rollover strategies. The values for the exchange rate hedge are almost unchanged for those strategies.

[Insert Table 5 about here]

As Table 6 shows, extending the planning horizon to two years does not alter the general result that the optimal hedge delivers significant improvements over heuristic alternatives that are employed in practice. The reduction in volatility is somewhat lower in this setting, but, with the expanded time horizon, this is not surprising. Therefore, all in all, the optimal hedge as derived in Corollary 1 can enhance corporate risk management when time risk is a factor, and it can do so in different markets, as well as for different time horizons.

[Insert Table 6 about here]

3.3 Results: Dependent Time and Price Risk

We now extend our analysis to allow for both customized non-linear payoff structures and dependence between time and price risk to assess the performance of the optimal strategy, given by Proposition 2. We compare it with a rollover and a long-term strategy, as well as the unconditional linear strategy given by Corollary 1 and the optimal linear strategy from Proposition 1. We exclude the comparison with both the no-hedge scenario and a deferred hedge, since a sophisticated non-linear optimal strategy is more likely to be considered by risk management officers with greater experience, who will already have some sort of policy in place.

[Insert Table 7 about here]

Table 7 presents the reductions in standard deviation for the base case. It can be seen that the non-linear hedge is indeed superior to all the other strategies, especially the long-term and rollover strategies. If the dependence level is high, volatility reductions can be as high as 60% for crude oil, 50% for copper, and 25% for gold when the reference point is the long-term strategy.

Compared to the unconditional strategy, the exotic hedge can yield improvements between 7% for gold and 28% for crude oil. Even when the optimal strategy replaces the optimal linear hedge, it can lead to reductions in volatility of up to 20% in the case of crude oil. Generally speaking, the higher the dependence between price and time risk, the better the non-linear strategy fares compared to the alternatives. While improvements for the currency hedge are somewhat lower, our findings still suggest there could be a market for customized over-the-counter derivatives tailored specifically to such exposures.

As in the previous section, we alter the hedging period to two months and two years, the results of which are presented in Tables 8 and 9, respectively. For the two-month horizon, the overall improvements achieved by the optimal strategy are less pronounced than in the base case. This is especially true for the exchange rate and gold, where the optimal linear strategy and the unconditional linear strategy perform almost as well. For copper and crude oil, the improvements are greater, especially when the dependence between prices and time risk is high. Then, the optimal strategy can reduce volatility by more than one-third compared to the long-term hedge.

[Insert Table 8 about here]

In the two-year case, the exotic hedge can lead to a substantial outperformance of the optimal linear strategy when dependence is high. For gold, the reduction in volatility for that case amounts to about 6% and reaches 16% for copper and even 22% for crude oil. When compared to the unconditional linear strategy, improvements can be as high as 44% and, compared to the long-term strategy, up to 68%. Even when dependence is only moderate, the optimal strategy can reduce volatility by up to 57%. Interestingly, the results for copper and oil also show that a rollover strategy could perform even better than considering the correct unconditional probability of an early payment and using the unconditional linear strategy if there is dependence between time risk and price risk.

[Insert Table 9 about here]

All in all, this illustrates two important results. First, a hedge as prescribed by the optimal strategy can substantially reduce volatility in profits, especially when time and price risk are strongly dependent and when the hedge horizon increases. Second, for shorter periods and with moderate dependence, the linear optimal strategy works quite well, so it may not be necessary to always include time and price risk dependence in one's hedging policy, particularly for the USD–euro exchange rate. This latter result can help explain the findings of Gay et al. (2002) and Huang et al. (2007), in which a large part of surveyed firms use only linear derivatives to hedge currency exposure.

4 Conclusions

Although the theory and practice of corporate hedging has been extensively discussed in the literature, uncertainty about the timing of cash flows has been largely ignored. Since time risk is prevalent in many actual situations, this paper incorporates it into the design of hedging strategies. Using a discrete-time model, the paper derives two dynamic hedging strategies that minimize the variance of profits: The first uses only linear contracts of different maturities. This linear hedging strategy is shown to be optimal if price risk and time risk are independent. In this case, only knowledge of the probabilities with respect to the timing of future cash flows is required and this strategy is relatively easy to implement. The second hedging strategy is more sophisticated and incorporates non-linear instruments, which can reduce profit variance even more in the case of dependent price and time risk. The key element of this strategy is the conditional probability of the timing of revenues, given all available price information. For the case of multiple products with multiple price risks, we show that these risks can be hedged independently, irrespective of the dependence structure between different time risks.

Our empirical study quantifies the effects of the derived hedging strategies for three commodities and the USD–euro exchange rate over a variety of scenarios. First, we show that the optimal hedge can deliver high levels of effectiveness in different markets. For a firm with a planning horizon of one year, a minimum reduction in standard deviation of 82% can be achieved in commodity markets, regardless of the probability of an early sale. For the USD–euro exchange rate, the minimum value is even 97%. For the shorter horizon of two months, the minimum reduction for crude oil increases to almost 89% while it is still 94% for the currency hedge. Even for a two-year horizon, variance reductions between 78% and 94% are possible for the four markets and different probabilities of an early sale.

We also show that the optimal hedge substantially outperforms several heuristic hedging strategies, regardless of the length of the planning horizon and the probability of an early sale. Especially when compared to a deferred hedge, that is, a hedging strategy that starts hedging only if exposure is known exactly, an optimal hedge can reduce the standard deviation of total profits by up to 95% in the one-year setting. In a two-month timeframe, even greater risk reductions are possible. Very substantial improvements over rollover or long-term hedging strategies are similarly possible but depend on the probabilities q. For $q = 0.5$, the improvements range from 6% for the exchange rate hedge to 25% for crude oil. We generally find that the higher the convenience yield risk of a commodity, the greater the resulting improvements of the optimal hedging strategy over the long-term and rollover strategies. Overall, our results indicate that the derived optimal strategy presents a superior way to hedge price risk when payment dates are uncertain. As we have shown analytically, this result often holds, even if q is not known exactly and is estimated with some degree of error.

Finally, we ask how closely hedging with linear instruments resembles the usage of non-linear exotic derivatives with an optimal payoff structure. For currency hedges with short hedge horizons and moderate dependence between price risk and time risk, the optimal linear strategy can suffice. Given that managers generally seem to be more familiar with linear derivatives and liquid futures markets are often available, hedging with linear instruments is a reasonable choice. For longer hedge horizons and strong dependence between price and time risk, however, tailor-made non-linear derivatives can lead to significant improvements in terms of risk reduction.

Appendix

Proof of Proposition 1: The minimization problem (7) is a dynamic one that has to be solved recursively. Therefore, the determination of the optimal hedging policy starts at date 1.

Date 1: According to Assumption 2, the use of derivatives does not change expected profits, which implies that variance minimization is equivalent to minimizing $E_0[\Pi^2]$. Therefore, the problem to be solved at date 1 can be written

$$\min_{h_{1,2}} E_1\left[\Pi^2\right]. \tag{22}$$

This optimization problem leads to the following first-order condition, which is also sufficient for a minimum:

$$\begin{aligned} & \tfrac{\partial E_1[\Pi^2]}{\partial h_{1,2}} = E_1[2\,\Pi\,(P_2 - F_{1,2})] = 0 \\ \Leftrightarrow \quad & Cov_1\left[\Pi, (P_2 - F_{1,2})\right] = 0. \end{aligned} \tag{23}$$

The equivalence of the two conditions in Equation (23) follows from the assumption that forwards earn zero expected profits. According to Equation (23), we require that profit be uncorrelated with the payoff of the forward contract. This is certainly the case if profit is not stochastic. Note that at date 1 time risk is no longer present. If $I = 1$, the sale has occurred and there is no price exposure.

If $I = 0$, operating profits are risky solely because of the uncertain price P_2. By choosing $h_{1,2} = -Q$, we completely eliminate this price risk.

In conclusion, buying $h_{1,2} = -Q(1 - I)$ forward contracts leads to a certain profit from the perspective of date 1, which means that the optimality condition is fulfilled.

Date 0: Given the optimal forward position at date 1, the firm's profit becomes

$$\begin{aligned} \Pi^* \;=\; & P_1\,Q\,I\,(1+r) + F_{1,2}\,Q\,(1-I) - C \\ & + (P_1 - F_{0,1})(1+r)\,h_{0,1} + (F_{1,2} - F_{0,2})\,h_{0,2} \end{aligned} \tag{24}$$

and the optimization problem can be written

$$\min_{h_{0,1},\ h_{0,2}} E_0[\,\Pi^{*\,2}]\,. \tag{25}$$

The minimization problem (25) leads to the following first-order conditions:

$$\begin{aligned} \tfrac{\partial E_0[\,\Pi^{*\,2}]}{\partial h_{0,1}} &= E_0[2\,\Pi^*\,(P_1 - F_{0,1})(1+r)] = 0 \\ \Leftrightarrow \qquad & Cov_0[\,\Pi^*, (P_1 - F_{0,1})(1+r)] = 0, \end{aligned} \tag{26}$$

$$\begin{aligned} \tfrac{\partial E_0[\,\Pi^{*\,2}]}{\partial h_{0,2}} &= E_0[2\,\Pi^*\,(F_{1,2} - F_{0,2})] = 0 \\ \Leftrightarrow \qquad & Cov_0[\,\Pi^*, (F_{1,2} - F_{0,2})] = 0, \end{aligned} \tag{27}$$

Solutions to the first-order conditions also satisfy the second-order conditions for a minimum. The solution is unique if the price changes of the two relevant forward contracts are not perfectly correlated, that is, if neither contract is redundant.

Let us define $\bar{h}_{0,1} := h_{0,1}/Q$, $\bar{h}_{0,2} := h_{0,2}/Q$, and $\bar{P}_1 = P_1(1+r)$. Then, using the more compact matrix notation, the optimality conditions (26) and (27) can be written

$$\begin{pmatrix} Cov_0\left[I(\bar{P}_1 - F_{1,2}) + \bar{P}_1\bar{h}_{0,1} + F_{1,2}(1+\bar{h}_{0,2})\,,\,\bar{P}_1\right] \\ Cov_0\left[I(\bar{P}_1 - F_{1,2}) + \bar{P}_1\bar{h}_{0,1} + F_{1,2}(1+\bar{h}_{0,2})\,,\,F_{1,2}\right] \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{28}$$

$$\Leftrightarrow \begin{pmatrix} Cov_0\left[I(\bar{P}_1 - F_{1,2})\,,\,\bar{P}_1\right] \\ Cov_0\left[I(\bar{P}_1 - F_{1,2})\,,\,F_{1,2}\right] \end{pmatrix} = -\begin{pmatrix} Var_P & Cov_{P,F} \\ Cov_{P,F} & Var_F \end{pmatrix} \begin{pmatrix} \bar{h}_{0,1} \\ 1+\bar{h}_{0,2} \end{pmatrix} \tag{29}$$

Splitting the covariance on the left-hand side of Equation (29) into two parts, we obtain

$$\begin{pmatrix} Cov_0\left[I\bar{P}_1\,,\,\bar{P}_1\right] \\ Cov_0\left[I\bar{P}_1\,,\,F_{1,2}\right] \end{pmatrix} - \begin{pmatrix} Cov_0\left[IF_{1,2}\,,\,\bar{P}_1\right] \\ Cov_0\left[IF_{1,2}\,,\,F_{1,2}\right] \end{pmatrix} \tag{30}$$

Note that for three random variables X, Y, and Z, $Cov(XY, Z) = E[X]Cov(Y, Z)+$

$E[Y]Cov(X,Z)+E[(X-E(X))(Y-E(Y))(Z-E(Z))]$.(Bohrnstedt and Goldberger (1969)). Based on this result, the difference between the two covariance terms above equals

$$\begin{pmatrix} Var_P & Cov_{P,F} \\ Cov_{P,F} & Var_F \end{pmatrix} \begin{pmatrix} q \\ -q \end{pmatrix} + \begin{pmatrix} Cov_{P,I} & -Cov_{P,I} \\ Cov_{F,I} & -Cov_{F,I} \end{pmatrix} \begin{pmatrix} F_{0,1}(1+r) \\ F_{0,2} \end{pmatrix} \tag{31}$$

$$+ \begin{pmatrix} CoSk_{P,P,I} - CoSk_{P,F,I} \\ CoSk_{P,F,I} - CoSk_{F,F,I} \end{pmatrix},$$

where we use the fact that $E_0[I] = q$, $E_0[\bar{P}_1] = F_{0,1}(1+r)$ and $E_0[F_{1,2}] = F_{0,2}$. Inserting the expressions from Equation (31) into Equation (29) and solving for $h_{0,1}$ and $h_{0,2}$ produces the result. □

Conditions leading to lower variance than for a long-term strategy or a rollover strategy if an estimate $\hat{q}$ is used to implement the optimal hedging policy: Because none of the strategies leaves any risk exposure in the second period, we can concentrate on the profit

$$\begin{aligned} \Pi^* \;&=\; P_1\, Q\, I\, (1+r) + F_{1,2}\, Q\, (1-I) - C \\ &\quad +(P_1 - F_{0,1})(1+r)\, h_{0,1} + (F_{1,2} - F_{0,2})\, h_{0,2}\,. \end{aligned} \tag{32}$$

Let us write the variance of Π^* as the expectation of the conditional variance, given I, plus the variance of the conditional expectation, given I:

$$Var_0(\Pi^*) = E_0\big[Var_0(\Pi^*|\, I)\,\big] + Var_0\big[E_0(\Pi^*|\, I)\,\big]. \tag{33}$$

Because we investigate the case of independent time risk and price risk, the conditional expectation of $\Pi^*|I$ equals the unconditional expectation. Since all derivatives contracts earn zero expected profits, this expectation does not depend on the hedging strategy and we can ignore the second term on the right-hand side of Equation

(33) for our considerations. Let us next calculate the expectation of the conditional variances for different hedging policies.

Optimal strategy that uses the estimate $\hat{q}$ instead of the true q: For this strategy, $h_{0,1} = -\hat{q}\,Q$ and $h_{0,2} = -(1-\hat{q})\,Q$. If $I = 0$, the conditional variance of Π^* becomes $Q^2\hat{q}^2 Var(P_1(1+r) - F_{1,2})$. If $I = 1$, we obtain a conditional variance $Q^2(1-\hat{q})^2 Var(P_1(1+r) - F_{1,2})$. Therefore, the expectation of the conditional variances equals $[(1-q)\,\hat{q}^2 + q(1-\hat{q})^2][Q^2 Var(P_1(1+r) - F_{1,2})]$.

Rollover strategy: The rollover strategy uses $h_{0,1} = -Q$ and $h_{0,2} = 0$. With this choice, the conditional variances become $Q^2 Var(P_1(1+r) - F_{1,2})$ $(I = 0)$ and zero $(I = 1)$, yielding an expectation of the conditional variances of $(1-q)[Q^2 Var(P_1(1+r) - F_{1,2})]$.

Long-term strategy: The long-term strategy uses $h_{0,1} = 0$ and $h_{0,2} = -Q$, which leads to conditional variances of zero $(I = 0)$ and $Q^2 Var(P_1(1+r) - F_{1,2})$ $(I = 1)$. The expectation of the conditional variances is $q[Q^2 Var(P_1(1+r) - F_{1,2})]$.

A comparison of the above variances yields the following conditions for a strategy using $\hat{q}$ instead of q leading to a lower profit variance than for the two alternatives $(1-q)\,\hat{q}^2 + q(1-\hat{q})^2 < 1-q$ (rollover) and $(1-q)\,\hat{q}^2 + q(1-\hat{q})^2 < q$ (long-term). Simplifying these inequalities finally delivers the conditions $\hat{q} > 2q-1$ (rollover) and $\hat{q} < 2q$ (long-term). □

Proof of Proposition 2: To prove the proposition, we have to solve the dynamic minimization problem (11). It has to be solved recursively, starting at date 1.

Date 1: At date 1, we consider the Lagrange function $E_1\,[\,\Pi^2 - \lambda_2\, g_2(P_2)]$ and solve the following functional optimization problem:

$$\min_{g_2(P_2)} E_1\,[\,\Pi^2 \; - \lambda_2\, g_2(P_2)]. \tag{34}$$

The first-order conditions for this problem, which are also sufficient for a minimum, can be written

$$\frac{\partial E_1[\,\Pi^2 - \lambda_2\, g_2(P_2)]}{\partial g_2(P_2)} = E_1\,[\,2\,\Pi - \lambda_2\,|\,P_2\,] = 0$$
$$\Leftrightarrow \quad E_1\,[\,\Pi|P_2\,] = \lambda_2/2, \quad \forall\; P_2 \in (0,\infty). \tag{35}$$

Since time risk is no longer present at date 1 and P_2 is the only remaining random variable, we can drop the expectation in Equation (35). Optimality requires that the profit be constant for all realizations of P_2. As we have seen in the proof of Proposition 1, selling $Q(1-I)$ forward contracts leads to a non-stochastic profit. Therefore, the corresponding payoff function fulfills the optimality conditions and $g_2^*(P_2) = -Q\,(1-I)\,(P_2 - F_{1,2})$.

Date 0: Given the optimal hedging policy at date 1, the firm's profit becomes

$$\Pi^* \;=\; P_1\,Q\,I\;(1+r) + F_{1,2}\,Q\,(1-I) - C + g_1(P_1, F_{1,2})(1+r). \tag{36}$$

At date 0, the relevant Lagrange function is $E_0\,[\,\Pi^{*\,2} - \lambda_1\, g_1(P_1, F_{1,2})]$ and we have to solve the following functional optimization problem:

$$\min_{g_1(P_1,F_{1,2})} \; E_0\,[\,\Pi^{*\,2} - \lambda_1\, g_1(P_1, F_{1,2})]. \tag{37}$$

The first-order conditions also provide sufficient conditions for this problem:

$$\frac{\partial E_0[\,\Pi^{*2} - \lambda_1\, g_1(P_1,F_{1,2})]}{\partial g_1(P_1,F_{1,2})} = E_0\,[\,2\,\Pi^* - \lambda_1\,|P_1, F_{1,2}\,] = 0$$
$$\Leftrightarrow \quad E_0\,[\,\Pi^*|P_1, F_{1,2}\,] = \lambda_1/2,\; \forall\;\; P_1 \in (0,\infty),\; F_{1,2} \in (0,\infty). \tag{38}$$

Equation (38) states that the expected profit, conditional on P_1 and $F_{1,2}$, must be the same for all values of P_1 and $F_{1,2}$. Note that the expectations in Equation (38) are taken with respect to time risk, that is, with respect to the random variable I. The conditions of Equation (38) are fulfilled if $g_1(P_1, F_{1,2}) = -Q\;\left[q_{\,|\,P_1\,F_{1,2}}\,P_1 \;+\; (1 - q_{\,|\,P_1,\,F_{1,2}})\,F_{1,2}\,(1+r)^{-1} \;-k\right]$, where $q_{\,|\,P_1\,F_{1,2}}$ denotes the conditional probability that a sale occurs at date 1, that is, the expectation of $I|P_1, F_{1,2}$. The constant k ensures that the expected payoff (with respect to P_1 and $F_{1,2}$) of the derivatives contract equals zero. Therefore, the specific value of k depends on the joint distribution of P_1 and $F_{1,2}$ and the dependence structure of price risk and time risk. □

Variances of different hedging strategies: Without loss of generality, set $Q = 1$, since Q is just a scaling factor that affects all hedging strategies similarly. Moreover, since all hedging strategies eliminate risk completely in the second period, we can start with the profit

$$\Pi^* \;=\; P_1\, I\, (1+r) + F_{1,2}\, (1-I) - C + g_1(P_1, F_{1,2})(1+r). \tag{39}$$

The variance of Π^* equals the expectation of the conditional variances, given P_1 and $F_{1,2}$, plus the variance of the conditional expectation, given $P1$ and $F_{1,2}$:

$$Var_0(\Pi^*) = E_0\big[Var_0(\Pi^* |\, P_1, F_{1,2})\,\big] + Var_0\big[E_0(\Pi^* |\, P_1, F_{1,2})\,\big]. \tag{40}$$

From Equation (39), we see that $Var_0(\Pi^* |\, P_1, F_{1,2}) = (P_1(1+r) - F_{1,2})^2 q_{|P}(1 - q_{|P})$, where $q_{|P}$ denotes the conditional expectation $E_0[\, I \,|\, P_1, F_{1,2}]$. Thus, the first term on the right-hand side of Equation (40) becomes $E_0\big[(P_1(1+r) - F_{1,2})^2 q_{|P}(1 - q_{|P})\big]$. This first component of the variance of Π^* does not depend on any derivative positions $g_1(P_1, F_{1,2})$ and is therefore identical for all strategies. However, the second component, $Var_0\big[E_0(\Pi^* |\, P_1, F_{1,2})\,\big]$, differs across different hedging policies and is calculated for each strategy separately, as follows.

1) Optimal hedge: For the optimal strategy, we have $g_1(P_1, F_{1,2}) = -[q_{|P}\, P_1 + (1 - q_{|P})\, F_{1,2}\, (1+r)^{-1} - k]$. Inserting this payoff function into Equation (39) shows that the conditional expectation is non-stochastic in this case, leading to $Var_0\big[E_0(\Pi^* |\, P_1, F_{1,2})\,\big] = 0$.

2) Optimal linear hedge: In this case, we use the payoff function from Proposition 1: $g_1(P_1, F_{1,2})(1+r) = [h^*_{0,1}\, (P_1 - F_{0,1})(1+r) \,+\, h^*_{0,2}\, (F_{1,2} - F_{0,2})]$. With this choice, we obtain a conditional expected profit of $(q_{|P} + h^*_{0,1})\, (P_1 - F_{0,1})\, (1+r) + (1 - q_{|P} + h^*_{0,2})\, (F_{1,2} - F_{0,2})$ plus some deterministic component. This deterministic component is irrelevant to the variance of the conditional expected profit.

3) Unconditional linear hedge: The relevant payoff function of derivatives is now

$g_1(P_1, F_{1,2})(1+r) = -[q\,(P_1 - F_{0,1})(1+r) \,+\, (1-q)\,(F_{1,2} - F_{0,2})]$. In this case, the required conditional expectation of the profit becomes $(q_{|P} - q)(P_1(1+r) - F_{1,2})$ plus deterministic components.

4) Rollover hedge: For the function $g_1(P_1, F_{1,2})(1+r) = -[(P_1 - F_{0,1})(1+r)]$, which characterizes the rollover strategy, the conditional expected profit equals $(q_{|P} - 1)(P_1(1+r) - F_{1,2})$ plus deterministic terms.

5) Long-term hedge: The long-term strategy with its payoff function $g_1(P_1, F_{1,2})(1+r) = -[F_{1,2} - F_{0,2}]$ leads to a conditional expectation of profits equal to $q_{|P}(P_1(1+r) - F_{1,2})$ plus deterministic terms.

6) Deferred hedge: A deferred hedge does not use any derivatives positions over the first period, that is, $g_1(P_1, F_{1,2}) = 0$. The corresponding stochastic component of the conditional expected profit is $q_{|P}\,(P_1 - F_{0,1})\,(1+r) + (1 - q_{|P})\,(F_{1,2} - F_{0,2})$.

7) No hedge: In this case, no derivatives positions are taken at date 1 or date 2, that is, $g_1(P_1, F_{1,2}) = 0$ and $g_2(P_2) = 0$. Therefore, the conditional expected profit equals $q_{|P}\,(P_1 - F_{0,1})\,(1+r) + (1 - q_{|P})\,(P_2 - F_{0,2})$. Moreover, the expectation of the conditional variance equals $E_0\big[(P_1(1+r) - P_{0,2})^2 q_{|P}(1 - q_{|P})\big]$. $\square$

References

Adam, T. (2009). Capital expenditures, financial constraints, and the use of options. *Journal of Financial Economics 92*(2), 238 – 251.

Adam, T. R. (2002). Risk management and the credit risk premium. *Journal of Banking & Finance 26*(2-3), 243 – 269.

Adam, T. R. and C. S. Fernando (2006). Hedging, speculation, and shareholder value. *Journal of Financial Economics 81*(2), 283 – 309.

Adam, T. R., C. S. Fernando, and J. M. Salas (2016). Why do firms engage in selective hedging? Evidence from the gold mining industry. *Journal of Banking & Finance*, forthcoming.

Adam-Müller, A. F. A. (1997). Export and hedging decisions under revenue and exchange rate risk: A note. *European Economic Review 41*(7), 1421 – 1426.

Adam-Müller, A. F. A. and I. Nolte (2011). Cross hedging under multiplicative basis risk. *Journal of Banking & Finance 35*(11), 2956 – 2964.

Adam-Müller, A. F. A. and A. Panaretou (2009). Risk management with options and futures under liquidity risk. *Journal of Futures Markets 29*(4), 297 – 318.

Allayannis, G. and J. P. Weston (2001). The use of foreign currency derivatives and firm market value. *Review of Financial Studies 14*(1), 43 – 276.

Anderson, R. W. and J.-P. Danthine (1980). Hedging and joint production: Theory and illustrations. *Journal of Finance 35*(2), 487 – 498.

Anderson, R. W. and J.-P. Danthine (1981). Cross hedging. *Journal of Political Economy 89*(6), 1182 – 1196.

Bartram, S. M. (2015). Corporate hedging and speculation with derivatives. Working paper, Warwick Business School.

Bartram, S. M., G. W. Brown, and J. Conrad (2011). The effects of derivatives on firm risk and value. *Journal of Financial & Quantitative Analysis 46*(4), 967 – 999.

Basak, S. and G. Chabakauri (2010). Dynamic mean-variance asset allocation. *Review of Financial Studies 23*(8), 2970 – 3016.

Benninga, S., R. Eldor, and I. Zilcha (1984). The optimal hedge ratio in unbiased futures markets. *Journal of Futures Markets 4*(2), 155 – 159.

Benninga, S. Z., R. Eldor, and I. Zilcha (1985). Optimal international hedging in commodity and currency forward markets. *Journal of International Money and Finance 4*(4), 537 – 552.

Bessembinder, H. (1991). Forward contracts and firm value: Investment incentive and contracting effects. *Journal of Financial & Quantitative Analysis 26*(4), 519 – 532.

Bodnar, G. M. and G. Gebhardt (1999). Derivatives usage in risk management by US and German non-financial firms: A comparative survey. *Journal of International Financial Management & Accounting 10*(3), 153 – 187.

Bodnar, G. M., G. S. Hayt, and R. C. Marston (1998). 1998 Wharton survey of financial risk management by us non-financial firms. *Financial Management 27*(4), 70 – 91.

Bohrnstedt, G. W. and A. S. Goldberger (1969). On the exact covariance of products of random variables. *Journal of the American Statistical Association 64*(328), 1439 – 1442.

Briys, E., M. Crouhy, and H. Schlesinger (1993). Optimal hedging in a futures market with background noise and basis risk. *European Economic Review 37*(5), 949 – 960.

Brown, G. W., P. R. Crabb, and D. Haushalter (2006). Are firms successful at selective hedging?. *Journal of Business 79*(6), 2925 – 2949.

Brown, G. W. and K. B. Toft (2002). How firms should hedge. *Review of Financial Studies 15*(4), 1283 – 1324.

Campbell, T. S. and W. A. Kracaw (1990). Corporate risk management and the incentive effects of debt. *Journal of Finance 45*(5), 1673 – 1686.

Carter, D. A., D. A. Rogers, and B. J. Simkins (2006). Does hedging affect firm value? Evidence from the US airline industry. *Financial Management 35*(1), 53 – 86.

Casassus, J. and P. Collin-Dufresne (2005). Stochastic convenience yield implied from commodity futures and interest rates. *Journal of Finance 60*(5), 2283 – 2331.

Chang, E. C. and K. P. Wong (2003). Cross-hedging with currency options and futures. *Journal of Financial & Quantitative Analysis 38*(3), 555 – 574.

Chowdhry, B. (1995). Corporate hedging of exchange risk when foreign currency cash flow is uncertain. *Management Science 41*(6), 1083 – 1090.

Cox, J. C., J. E. Ingersoll, and S. A. Ross (1981). The relation between forward prices and futures prices. *Journal of Financial Economics 9*(4), 321 – 346.

Culp, C. L. and M. H. Miller (1995). Metallgesellschaft and the economics of synthetic storage. *Journal of Applied Corporate Finance 7*(4), 62 – 76.

DeMarzo, P. M. and D. Duffie (1995). Corporate incentives for hedging and hedge accounting. *Review of Financial Studies 8*(3), 743 – 771.

Eaker, M. R. and D. Grant (1985). Optimal hedging of uncertain and long-term foreign exchange exposure. *Journal of Banking & Finance 9*(2), 221 – 231.

Froot, K. A., D. S. Scharfstein, and J. C. Stein (1993). Risk management: Coordinating corporate investment and financing policies. *Journal of Finance 48*(5), 1629 – 1658.

Gay, G. D., J. Nam, and M. Turac (2002). How firms manage risk: The optimal mix of linear and non-linear derivatives. *Journal of Applied Corporate Finance 14*(4), 82 – 93.

Huang, P., H. E. Ryan, and R. A. Wiggins (2007). The influence of firm- and ceo-specific characteristics on the use of nonlinear derivative instruments. *Journal of Financial Research 30*(3), 415 – 436.

Johnson, L. L. (1960). The theory of hedging and speculation in commodity futures. *Review of Economic Studies 27*, 139 – 151.

Kerkvliet, J. and M. H. Moffett (1991). The hedging of an uncertain future foreign currency cash flow. *Journal of Financial & Quantitative Analysis 26*(4), 565 – 578.

Korn, O. (2010). How firms should hedge: An extension. *Journal of Futures Markets 30*(9), 834 – 845.

Korn, O. and M.-O. Rieger (2016). Hedging with regret. Working paper, Göttingen University.

Mahul, O. (2002). Hedging in futures and options markets with basis risk. *Journal of Futures Markets 22*(1), 59 – 72.

Mahul, O. and J. D. Cummins (2008). Hedging under counterparty credit uncertainty. *Journal of Futures Markets 28*(3), 248 – 263.

Morellec, E. and J. Smith, Clifford W. (2007). Agency conflicts and risk management. *Review of Finance 11*(1), 1 – 23.

Moschini, G. and H. Lapan (1995). The hedging role of options and futures under joint price, basis, and production risk. *International Economic Review 36*(4), 1025 – 1049.

Rolfo, J. (1980). Optimal hedging under price and quantity uncertainty : The case of a cocoa producer. *Journal of Political Economy 88*(1), 100 – 116.

Ross, M. P. (1996). Corporate hedging: What, why and how? Working paper, University of California, Berkely.

Schwartz, E. S. (1997). The stochastic behavior of commodity prices: Implications for valuation and hedging. *Journal of Finance 52*(3), 922 – 973.

Smith, C. W. and R. M. Stulz (1985). The determinants of firms' hedging policies. *Journal of Financial & Quantitative Analysis 20*(4), 391 – 405.

Stulz, R. (2013). How companies can use hedging to crate shareholder value. *Journal of Applied Corporate Finance 25*(4), 21 – 29.

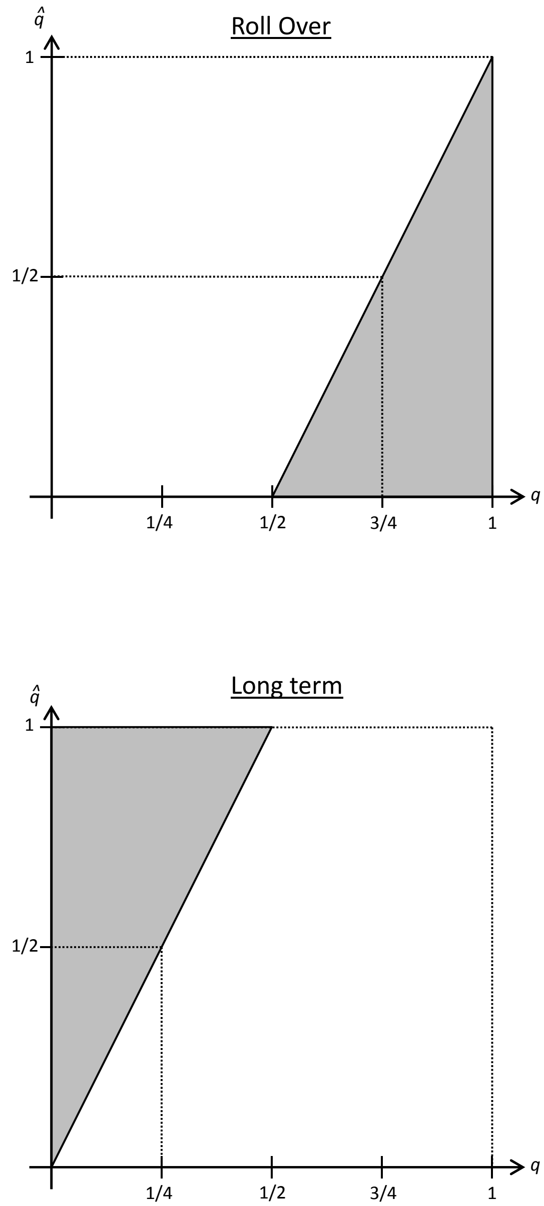

Figure 1: Optimality with misstated q
This figure depicts combinations of the true probability (q) of a sale occurring at date 1 and the probability used for implementing the optimal hedging strategy ($\hat{q}$) that lead to a better (worse) performance of the implemented optimal strategy in comparison with heuristic alternative strategies. The light area refers to combinations where the implemented optimal strategy leads to lower variance and the dark area to combinations where it leads to higher variance. The top and bottom panels of the figure compare the implemented optimal strategy with the long-term and rollover strategies, respectively.

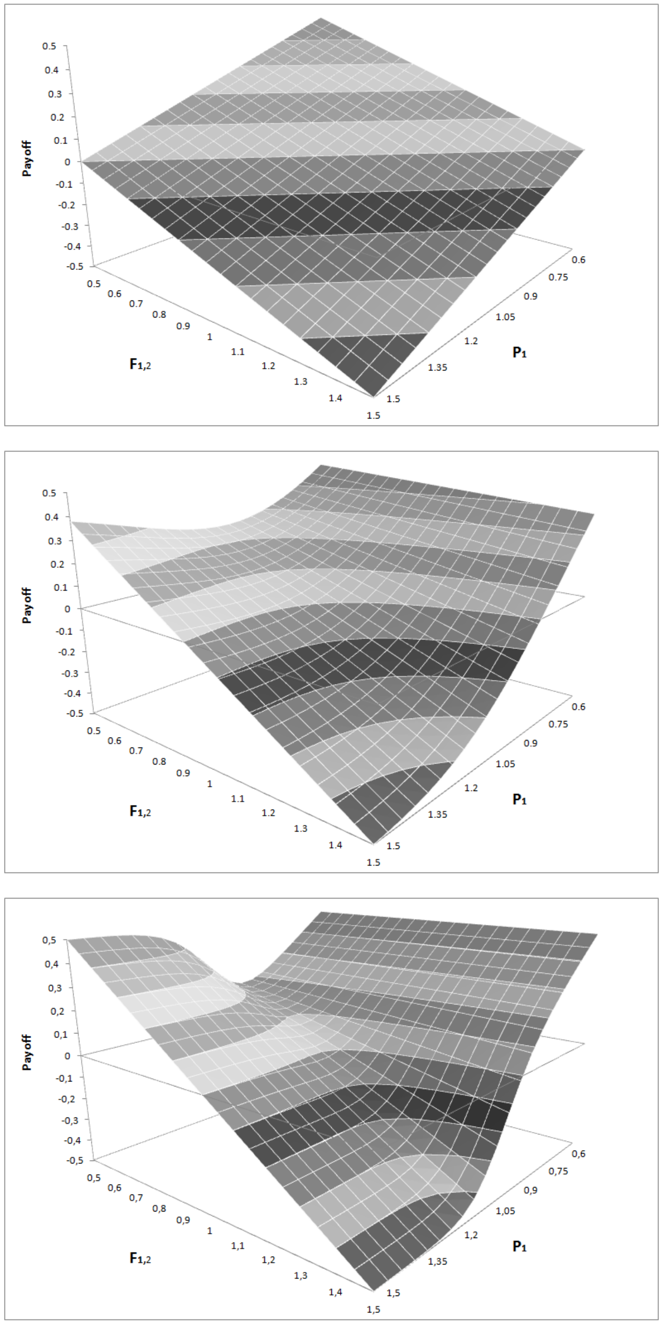

Figure 2: Optimal payoff function
This figure shows the optimal payoff functions of exotic derivatives contracts written on the spot price P_1 and the futures price $F_{1,2}$ for different levels of dependence between price risk and time risk. The top part of the figure refers to the case of independent risks ($\beta = 0$), the middle part to a moderate level of dependence ($\beta = 4$), and the bottom part to strong dependence ($\beta = 8$).

Table 1: Hedging effectiveness: Results for the gold, copper, crude oil, and FX markets over a one-year planning horizon with q as the probability of a sale taking place at $t = 1$.

q	**Gold**	**Copper**	**Crude Oil**	**\$/€**
0.1	96.87	94.43	90.85	97.15
0.2	95.70	92.56	87.47	97.06
0.3	94.91	91.20	85.23	97.04
0.4	94.37	88.48	83.72	97.02
0.5	94.03	89.65	82.84	97.02
0.6	93.20	89.45	82.62	97.53
0.7	94.05	89.68	83.10	97.02
0.8	94.54	90.54	84.63	97.03
0.9	95.68	92.51	87.95	97.09

Table 2: Hedging effectiveness: Results for the gold, copper, crude oil, and FX markets over a two-month planning horizon with q as the probability of a sale taking place at $t = 1$.

q	**Gold**	**Copper**	**Crude Oil**	**\$/€**
0.1	97.84	95.37	94.11	96.69
0.2	97.06	93.63	91.92	95.47
0.3	96.55	92.47	90.46	94.65
0.4	96.22	91.68	88.87	94.09
0.5	96.04	91.18	88.68	93.76
0.6	96.01	90.17	88.98	93.65
0.7	96.15	91.24	88.89	93.82
0.8	96.53	92.00	89.95	94.36
0.9	97.31	93.68	92.09	95.56

Table 3: Hedging effectiveness: Results for the gold, copper, crude oil, and FX markets over a two-year planning horizon with q as the probability of a sale taking place at $t = 1$.

q	**Gold**	**Copper**	**Crude Oil**	**\$/€**
0.1	96.42	92.87	88.42	88.53
0.2	95.05	90.20	84.25	84.43
0.3	94.11	88.39	81.58	81.79
0.4	93.45	84.61	79.87	80.09
0.5	93.01	86.33	78.98	79.16
0.6	92.80	86.05	78.90	79.01
0.7	92.89	86.34	79.75	83.66
0.8	93.39	87.45	81.84	81.70
0.9	94.68	90.04	85.98	85.73

Table 4: Reduction in standard deviation over a one-year planning horizon when switching from the respective strategies to the optimal hedge given in Corollary 1. Values are in percent and q denotes the likelihood of a sale taking place at the end of the first period.

q	**Long Term**	**Rollover**	**Deferred**	**q**	**Long Term**	**Rollover**	**Deferred**
		$/€				*Gold*	
0.1	0.76	33.26	89.01	0.1	1.75	49.44	95.29
0.2	1.69	19.77	85.47	0.2	3.81	33.97	93.73
0.3	2.84	13.06	83.46	0.3	6.29	24.51	92.84
0.4	4.32	9.00	82.41	0.4	9.30	17.94	92.35
0.5	6.27	6.27	82.12	0.5	13.08	13.08	92.21
0.6	9.00	4.32	82.53	0.6	17.94	9.30	92.38
0.7	13.06	2.85	83.69	0.7	24.51	6.29	92.88
0.8	19.77	1.69	85.77	0.8	33.97	3.81	93.79
0.9	33.26	0.76	89.33	0.9	49.44	1.75	95.35
		Copper				*Crude Oil*	
0.1	2.98	59.35	91.44	0.1	4.14	64.97	86.72
0.2	6.35	44.49	88.69	0.2	8.66	51.08	82.58
0.3	10.22	34.20	87.15	0.3	13.63	40.80	82.87
0.4	14.69	26.32	86.37	0.4	19.15	32.45	79.31
0.5	19.98	19.98	86.37	0.5	25.35	25.35	79.18
0.6	26.32	14.69	86.56	0.6	32.45	19.15	79.87
0.7	34.20	10.22	87.51	0.7	40.80	13.63	81.41
0.8	44.49	6.35	89.16	0.8	51.08	8.66	83.97
0.9	59.35	2.98	91.91	0.9	64.97	4.14	88.11

Table 5: Reduction in standard deviation over a two-month planning horizon when switching from the respective strategies to the optimal hedge given in Corollary 1. Values are in percent and q denotes the likelihood of a sale taking place at the end of the first period.

q	Long Term	Roll Over	Deferred	q	Long Term	Roll Over	Deferred
		\$/€				*Gold*	
0.1	0.78	33.67	95.30	0.1	0.53	26.91	97.20
0.2	1.72	20.08	93.74	0.2	1.19	15.11	96.27
0.3	2.90	13.29	92.83	0.3	2.01	9.69	95.73
0.4	4.40	9.17	92.34	0.4	3.08	6.56	95.43
0.5	6.40	6.40	92.19	0.5	4.52	4.52	95.34
0.6	9.17	4.40	92.35	0.6	6.56	3.08	95.44
0.7	13.29	2.90	92.85	0.7	9.69	2.01	95.74
0.8	20.08	1.72	93.76	0.8	15.11	1.19	96.28
0.9	33.67	0.78	95.32	0.9	26.91	0.53	97.21
		Copper				*Crude Oil*	
0.1	3.77	63.41	93.21	0.1	5.00	67.99	91.38
0.2	7.93	49.20	90.97	0.2	10.32	54.79	88.55
0.3	12.57	38.89	89.68	0.3	16.00	44.71	86.94
0.4	17.78	30.63	88.99	0.4	22.12	36.24	86.08
0.5	23.78	23.78	88.76	0.5	28.81	28.81	85.85
0.6	30.63	17.78	89.04	0.6	36.24	22.12	86.18
0.7	38.89	12.57	89.77	0.7	44.71	16.00	87.12
0.8	49.20	7.93	91.09	0.8	54.79	10.32	88.80
0.9	63.41	3.77	93.32	0.9	67.99	5.00	91.63

Table 6: Reduction in standard deviation over a two-year planning horizon when switching from the respective strategies to the optimal hedge given in Corollary 1. Values are in percent and q denotes the likelihood of a sale taking place at the end of the first period.

q	**Long Term**	**Roll Over**	**Deferred**	**q**	**Long Term**	**Roll Over**	**Deferred**
		$/€				*Gold*	
0.1	0.74	32.85	84.17	0.1	2.21	53.85	93.92
0.2	1.65	19.45	79.23	0.2	4.77	38.49	91.93
0.3	2.78	12.85	76.51	0.3	7.79	28.54	90.78
0.4	4.23	8.83	75.15	0.4	11.40	21.34	90.18
0.5	6.15	6.15	74.84	0.5	15.80	15.80	90.01
0.6	8.83	4.23	75.49	0.6	21.34	11.40	90.24
0.7	12.85	2.78	77.16	0.7	28.54	7.79	90.89
0.8	19.45	1.65	88.09	0.8	38.49	4.77	92.07
0.9	32.85	0.74	85.06	0.9	53.85	2.21	94.07
		Copper				*Crude Oil*	
0.1	2.94	59.14	87.93	0.1	3.66	62.91	82.61
0.2	6.28	44.25	84.22	0.2	7.71	48.61	77.47
0.3	10.11	33.97	82.24	0.3	12.24	38.28	74.86
0.4	14.55	26.11	81.33	0.4	17.36	30.07	73.77
0.5	19.79	19.79	81.21	0.5	23.23	23.23	73.82
0.6	26.11	14.55	81.88	0.6	30.07	17.36	74.87
0.7	33.97	10.11	83.28	0.7	38.28	12.24	76.93
0.8	44.25	6.28	85.58	0.8	48.61	7.71	80.21
0.9	59.14	2.94	89.31	0.9	62.91	3.66	85.40

Table 7: Reduction in standard deviation over a one-year planning horizon when switching from the respective strategies to the optimal hedge under dependent price and time risk given in Proposition 2. Values are in percent.

Dep.	**Opt Lin**	**Uncon Lin**	**Long Term**	**Rollover**	**Dep**	**Opt Lin**	**Uncon Lin**	**Long Term**	**Rollover**
		\$/€					*Gold*		
Moderate	0.17	1.31	9.15	5.78	Moderate	0.62	2.13	18.68	11.24
High	0.60	4.73	13.69	7.48	High	2.30	7.10	25.77	12.84
		Copper					*Crude Oil*		
Moderate	4.11	9.66	37.86	14.98	Moderate	9.50	13.98	47.41	21.93
High	10.74	22.40	50.32	19.44	High	20.02	28.14	60.01	28.80

Table 8: Reduction in standard deviation over a two-month planning horizon when switching from the respective strategies to the optimal hedge under dependent price and time risk given in Proposition 2. Values are in percent.

Dep.	**Opt Lin**	**Uncon Lin**	**Long Term**	**Rollover**	**Dep**	**Opt Lin**	**Uncon Lin**	**Long Term**	**Rollover**
		\$/€					*Gold*		
Moderate	0.03	0.21	6.81	6.40	Moderate	0.04	0.53	6.81	3.18
High	0.11	0.84	7.59	6.78	High	0.14	2.02	9.79	2.87
		Copper					*Crude Oil*		
Moderate	1.52	2.27	28.35	22.35	Moderate	3.70	3.84	30.19	32.83
High	4.96	7.40	34.61	23.30	High	11.29	11.55	35.50	38.56

Table 9: Reduction in standard deviation over a two-year planning horizon when switching from the respective strategies to the optimal hedge under dependent price and time risk given in Proposition 2. Values are in percent.

Dep.	**Opt Lin**	**Uncon Lin**	**Long Term**	**Rollover**	**Dep**	**Opt Lin**	**Uncon Lin**	**Long Term**	**Rollover**
		\$/€					*Gold*		
Moderate	0.31	2.45	11.43	5.21	Moderate	1.81	5.43	28.39	12.50
High	1.09	8.40	18.89	8.78	High	5.70	15.72	40.73	17.42
		Copper					*Crude Oil*		
Moderate	6.52	24.99	54.87	11.65	Moderate	13.10	24.37	57.39	19.60
High	16.40	44.47	68.40	21.78	High	22.24	37.07	67.45	26.92

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