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Abstract

We utilize Bayesian model averaging to estimate a stochastic discount factor (SDF) for single-stock options. A Bayesian model averaging SDF outperforms reduced-form benchmark models in-sample and out-of-sample in pricing option return anomalies and portfolios. We document that the SDF is dense in characteristics with the implied-realized volatility spread, option return momentum, and jump risk emerging as the most likely included factors. Noteworthy, we find that (i) our results remain largely robust after controlling for transaction costs and (ii) characteristics linked to behavioral biases gain in importance for options with high retail trading volume.

JEL classification: G12, G14, C11, C12, C52, C53

Keywords: Equity options; Option factor models; Asset pricing; Bayesian model averaging

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Abstract

We utilize Bayesian model averaging to estimate a stochastic discount factor (SDF) for single-stock options. A Bayesian model averaging SDF outperforms reduced-form benchmark models in-sample and out-of-sample in pricing option return anomalies and portfolios. We document that the SDF is dense in characteristics with the implied-realized volatility spread, option return momentum, and jump risk emerging as the most likely included factors. Noteworthy, we find that (i) our results remain largely robust after controlling for transaction costs and (ii) characteristics linked to behavioral biases gain in importance for options with high retail trading volume.

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1. Introduction

The rapid expansion of the options market, marked by explosive growth in contract volumes, underscores its thriving role in the financial landscape: According to the Option Clearing Corporation, the average daily trading volume in the U.S. options market soared to 44.2 million contracts in 2023, a staggering increase from just 11.4 million contracts in 2007. This surge in market activity has been paralleled by a growing academic interest in understanding the characteristics explaining the cross-section of individual equity option returns. Consequently, these trends have led to the discovery of a vast set of option return anomalies, a phenomenon previously coined as the "factor zoo" in the stock universe (Cochrane, 2011).

While the focus of previous academic research has been on identifying return anomalies, less emphasis has been placed on utilizing a combination of these anomalies as a multi-factor model to price options. Exceptions include (a) Horenstein, Vasquez, & Xiao (2022) who propose a latent three-factor structure captured by an option market factor, a factor based on the difference between implied and realized volatility, and a factor based on the volatility of implied volatility, (b) Zhan, Han, Cao, & Tong (2022) who price nine out of ten option return anomalies using a two-factor model that includes factors constructed by sorting on underlying stock liquidity and idiosyncratic volatility, and (c) Tian & Wu (2023) who suggest a five-factor model in which they add option momentum and the difference between implied and realized volatility to three factors that capture the risks and costs of market making (volatility risk, jump risk, and hedging costs). However, such low-dimensional factor models may suffer from potentially omitted variable bias and often fail to capture the full spectrum of influencing factors, leading to incomplete estimations of option returns and leftover abnormal returns. Moreover, not only are the competing models relatively recent, but they are also quite different from one another, indicating that no proposed model has established itself as the dominant yet.

In response to these shortcomings of low-dimensional factor models for individual equity option returns, our study utilizes the Bayesian model averaging (BMA) approach, introduced by Bryzgalova, Huang, & Julliard (2023), to estimate a SDF from a large cross-section of potentially relevant traded and non-traded factors. This approach is especially useful as it handles traditional weak points of the generalized method of moments (GMM), namely the presence of weak and level factors, while efficiently selecting true pricing sources. For our empirical analysis, we assemble a broad collection of factors to price the cross-section of call and put options, including 30 traded option factors (including the discovered anomalies from previous option research), 15 non-traded factors, as well as six widely used stock market factors. Our sample period is from 1996 to 2021. To construct the factors and test assets, we use monthly delta-hedged returns from the OptionMetrics database with a daily hedging schedule to refrain from options' sensitivities to the price movements of the respective underlying.

We identify several key option factors with high posterior probabilities of being included in the SDF that prices both the traded factor set and 25 long portfolios constructed by independently sorting on options' open-dollar interest and the implied minus realized volatility spread. For both calls and puts, the BMA-SDF selects (i) the difference between implied and realized volatility, (ii) option return momentum, and (iii) jump risk as the most important characteristics for all imposed levels of shrinkage. Nevertheless, we also provide evidence that the true SDF is dense rather than sparse. First, the average model dimension is large for the models chosen by the BMA approach. Second, no dominant model arises. Rather, there are many models with similar posterior probabilities. Third, when compared with traditional factor models, the BMA approach by Bryzgalova, Huang, & Julliard (2023) demonstrates superior pricing performance, even though some benchmark models share the most-likely to be included factors (i) to (iii), providing evidence that other factors exhibit relevant pricing information too.

Our results are consistent in both in-sample and out-of-sample tests across different cross-sections of assets and time periods. For the cross-sectional out-of-sample performance, we assess the different models' capability of pricing 17 industry-sorted option portfolios and 26 additional option return anomalies as identified by Goyal & Saretto (2024) that are not featured in stand-alone academic papers. For the out-of-sample tests in the time-series dimension, we estimate the BMA-SDF over the first (second) half of our sample period and then evaluate its pricing performance over the second (first) half. In the context of this analysis, we observe that the composition of factors with the highest posterior model probabilities changes over time: Whereas general mispricing, as captured by the difference between implied and realized volatility, is by far the most dominant factor in the first subperiod, more risk-related option factors such as jump risk gain in importance during the second subperiod. In addition, we introduce new "best factor" models, created by selecting factors with the highest posterior probabilities.[1]

Detzel, Novy-Marx, & Velikov (2023) show that failing to adjust asset pricing models for transaction costs obscures the distinction between true risk premia and unattainable paper profits, with estimated SDFs and optimal portfolios falsely concentrating on the latter. This might be especially relevant for options markets as transaction costs are particularly high compared to other asset classes (e.g., Muravyev & Pearson, 2020; Goyal & Saretto, 2024). To address this issue, we calculate factor and test asset returns net of transaction costs and repeat our main analysis. Factors based on option momentum and the difference between implied and realized volatility capture genuine risk premia beyond just limits to arbitrage, while the jump risk factor vanishes as a likely candidate of the true SDF due to trading on options with high trading costs. The pricing performance of the BMA-SDF remains superior over low-dimensional benchmark models when accounting for transaction costs.

Bryzgalova, Pavlova, & Sikorskaya (2023) document a rising share of retail traders for equity options over the last years. To assess whether a higher concentration of retail trading in certain options contracts influences the composition of the SDF that prices these options, we use signed volume data from four NASDAQ options exchanges to identify options with high and low small customer volumes. After constructing our option factors and test assets with above- and below-median retail volume, we repeat our analysis and find that the spread between implied-minus-realized volatility, a measure related to general option mispricing, and the log price of the underlying, a characteristics related to investor inattention (Boulatov

[1]The superior out-of-sample pricing performance of even the factor model with only four factors underscores the BMA approach's utility in factor selection. However, the further improvements in the pricing power of the best ten-factor model show that there are limitations in the model reduction because of the dense nature of the true SDF.

et al., 2022), are more pronounced in the high-retail investor subsample. This finding is consistent with the notion that option prices are affected by behavioral biases to which less sophisticated investors are more susceptible.

Related literature

First and foremost, our paper relates to Bryzgalova, Huang, & Julliard (2023), who introduce a Bayesian method for model estimation, selection, and averaging for traded and non-traded factors, allowing for the presence of both weak and strong factors. The crucial result of the authors' empirical analysis is that a dense space of factors characterizes the SDF for stock returns. Hence, the introduction of a BMA-SDF serves as the optimal approach to aggregate factors and thereby spans the "true" SDF of equity returns characterized by a multitude of factors proxying for similar risks. Dickerson, Julliard, & Mueller (2023) implement the Bayesian method introduced by Bryzgalova, Huang, & Julliard (2023) to price corporate bond returns using a joint sample of bond and equity factors. Similar to the SDF for stock returns, they document a dense factor structure of the SDF that prices bond returns. To our knowledge, we are the first to apply Bayesian methods in the context of assessing linear factor models for individual equity options. In line with the findings for stocks and bonds, we document a dense factor structure of the SDF in the options market and a superior pricing performance by the BMA-SDF compared to reduced-form benchmark models.

Turning to the literature on option returns, our paper contributes to previous work that analyses characteristics explaining option returns in the cross-section. Up to now, the existing literature has established two sets of characteristics with explanatory power for the cross-section of option returns. First, characteristics related to market makers' hedging capabilities and incurred risks drive option returns as these liquidity providers require compensation for higher incurred hedging costs. For instance, Cao & Han (2013) document that options on stocks with high idiosyncratic volatility yield lower returns as option end-users display high demand for these options, and option dealers as counterparties in the transactions must more frequently adjust their delta-hedge positions for the options on stocks with high idiosyncratic volatility. Tian & Wu (2023) highlight the relevance of primary risks for option market makers, namely delta-hedging costs, volatility risk, and jump risk, to explain option returns.[2] Second, end-user demand for options with specific characteristics can affect option prices and, therefore, results in cross-sectional return predictability. For instance, Frazzini & Pedersen (2022) argue that investors are willing to pay a premium for options with higher embedded leverage and show that these contracts consequently display lower returns. In addition, several studies point out behavioral aspects that impact options prices. Byun & Kim (2016) stress the importance of investors' gambling preferences leading to an overvaluation of options on lottery-like stocks. Boulatov et al. (2022) demonstrate that due to limited investor attention, options on low-priced stocks are overpriced as investors incorrectly consider them bargains.[3] We relate to this strand of literature by documenting that

[2]Also, other general costs to market-making drive option prices: Christoffersen et al. (2018) find a positive liquidity premium for single-name options with high spreads to compensate market makers that are, on average, long in these contracts.

[3]Other option and underlying stock characteristics with pricing power are less clearly assignable to one of the two categories outlined above. For example, Goyal & Saretto (2009) hypothesize that the difference between implied and historically realized volatility is related to investor overreaction, however, this characteristic might also proxy for a volatility risk premium and, importantly, any (residual) option mispricing (Tian & Wu, 2023). Zhan et al. (2022) find a multitude of profitable option strategies by sorting on stock-level characteristics and cannot identify an unambiguous explanation for these return anomalies. Furthermore, Heston, Jones, Khorram, Li, & Mo (2023) do not find a clear risk- or behavioral-based explanation for the momentum effects in option returns. In this context, Käfer, Moerke, & Wiest (2023) show that single option momentum stems from option *factor* momentum, indicating that persistent factor risk premia, as well as

pricing factors linked to behavioral biases and mispricing gain in importance for explaining the returns of options with high retail volume.

Finally, recent work such as Horenstein et al. (2022) or Tian & Wu (2023) attempt to explain the predictability of option returns in the cross-section by introducing linear factor models for the options market. Goyal & Saretto (2024) can explain the returns of 46 option trading strategies using an IPCA (instrumented principal component analysis) model. Our paper contributes to the existing option factor models by identifying option factors that are most likely to be part of the SDF using Bayesian methods. We show that the factors identified by the Bayesian method outperform existing reduced-form benchmark models in out-of-sample tests. However, we also find that the BMA approach outperforms any reduced-form factor model, in line with our finding that the factor space in the options market is dense.

The remainder of this paper is organized as follows. Section 2 provides a brief overview of the Bayesian methods applied in our empirical analyses. Section 3 summarizes our data collection, option return definition, and option factor construction. Section 4 presents our main empirical results and Section 5 includes additional analyses on the impact of transaction costs and retail trading as well as robustness checks. Section 6 concludes.

persistent variation across the factor premia, drive option momentum.

2. Methodology

We adapt the methodology of Bryzgalova, Huang, & Julliard (2023) and Dickerson et al. (2023) to provide a Bayesian analysis of linear stochastic discount factor models in the single-name equity options market. In this section, we shortly summarize the methodology. For a detailed treatment, we refer the reader to Bryzgalova, Huang, & Julliard (2023) and Dickerson et al. (2023).

We begin by considering a linear factor model for the stochastic discount factor (SDF). In general, let $\mathbb{E}[X] \equiv \mu_X$ be the unconditional expectation of the random variable X. Furthermore, let 1_N (0_N) denote a N-dimensional vector of ones (zeros) and $R_t = (R_{1,t}, \ldots, R_{N,t})^T \in \mathbb{R}^N$ represent the time-t returns of N test assets. Next, we consider a set of K tradable ($f_t^{(1)} \in \mathbb{R}^{K_1}$) and non-tradable factors ($f_t^{(2)} \in \mathbb{R}^{K_2}$), where $K = K_1 + K_2$. A linear SDF has the form $M_t = 1 - (f_t - \mathbb{E}[f_t])^T \lambda_f$, where $\lambda_f \in \mathbb{R}^K$ is the vector of market prices of risk for the factors. Under no-arbitrage, $M_t R_t = 0_N$, and expected returns are expressed as $\mu_R = \mathbb{E}[R_t] = C_f \lambda_f$ with C_f being the covariance matrix between R_t and f_t. Define $C = (1_N, C_f), \lambda^T = (\lambda_c, \lambda_f^T)$ with λ_c being average mispricing, and $\alpha \in \mathbb{R}^N$ being a vector of pricing errors in excess of λ_c. Then, market prices of risk, λ_f, can be estimated by running the following cross-sectional regression

$$\mu_R = \lambda_c 1_N + C_f \lambda_f + \alpha = C\lambda + \alpha. \tag{1}$$

We follow Bryzgalova, Huang, & Julliard (2023) and Dickerson et al. (2023) and specify prior and posterior probabilities for factors, returns, and the average pricing error. Specifically, the time-series of the union of factors and returns, $Y_t = f_t^{(2)} \cup R_t, Y_t \in \mathbb{R}^{p+K_2}$, is multivariate Gaussian with mean μ_Y and variance matrix Σ_Y. By utilizing the usual diffusive prior for the time-series parameters (μ_Y, Σ_Y) as $\pi(\mu_Y, \Sigma_Y) \propto |\Sigma_Y|$ yields Normal-inverse Wishart posteriors. Assuming that average pricing errors α are following a Normal distribution with mean-zero and variance matrix $\sigma^2\Sigma_R$, the cross-sectional likelihood is given as

$$p(\text{data}|\lambda, \sigma^2) = (2\pi\sigma^2)^{-\frac{N}{2}} |\Sigma_R|^{-\frac{1}{2}} \exp\left(-\frac{1}{2\sigma^2}(\mu_r - C\lambda)^T \sigma_R^{-1}(\mu_R - C\lambda)\right), \tag{2}$$

where the expected risk premia, μ_R, and the factor loadings, $C = (1_N, C_f)$ constitute "data" in Equation (2). As the goal is to arrive at a posterior distribution for different models for the SDF, Bryzgalova, Huang, & Julliard (2023) and Dickerson et al. (2023) specify the following prior for the risk prices. Let $\gamma^T = (\gamma_0, \ldots, \gamma_K), \gamma_j \in \{0, 1\} \forall j = 1, \ldots, K$ be a vector of binary variables for denoting a selection of factors for the SDF, i.e., $\gamma_j = 1$ if the j-th factor is included in the SDF, otherwise $\gamma_j = 0$. Bryzgalova, Huang, & Julliard (2023) propose to use a continuous spike-and-slab mixture prior instead of flat priors, $\pi(\lambda, \sigma^2, \gamma, \omega)$. They motivate this choice by the possible presence of weak factors which can render the definition of posterior probabilities undefinable for flat priors. $\pi(\lambda, \sigma^2, \gamma, \omega)$ is given as

$$\begin{aligned} \pi(\lambda, \sigma^2, \gamma, \omega) &= \pi(\lambda|\sigma^2, \gamma) \times \pi(\sigma^2) \times \pi(\gamma|\omega) \times \pi(\omega), \\ \lambda_j|\gamma_j, \sigma^2 &\sim \mathcal{N}(0, r(\gamma_j)\psi_j\sigma^2), \end{aligned} \tag{3}$$

where $r(\gamma_j)$ introduces the spike-and-slab prior. If the j-th factor should be included in the SDF, $r(\gamma_j = 1) = 1$, and the prior distribution for λ_j is diffuse with mean zero. On the other hand, if the j-th factor should not be included in the SDF, $r(\gamma_j = 0) = r \ll 1$, and the prior is concentrated at zero. ψ_j in Equation (3) penalizes factors that are likely caused by identification failure and is determined data-driven as $\psi_j = \psi \times \tilde{\rho}_j^T \tilde{\rho}_j$ with $\tilde{\rho}_j = \rho_j - \frac{1}{N}\sum_{i=1}^{N} \rho_{i,j} \times 1_N$, where $\rho_j \in \mathbb{R}^N$ is a vector of correlation coefficients between factor j and the test assets, and $\psi \in \mathbb{R}_+$ is a tuning parameter controlling the degree of shrinkage across all factors. ψ has an economic interpretation as it is linked to the expected prior Sharpe ratio (SR) which can be achieved with all factors. It holds that $\mathbb{E}_\pi(SR_f^2|\sigma^2) = \frac{1}{2}\psi\sigma^2 \sum_{k=1}^{K} \tilde{\rho}_k^T \tilde{\rho}_k$ for $r \to 0$. $\pi(\omega)$ in Equation (3) serves two purposes. It yields a way to sample across the space of all potential models and it incorporates the prior on the sparsity of the true model. Bryzgalova, Huang, & Julliard (2023) and Dickerson et al. (2023) follow the previous literature and use

$$\pi(\gamma_j = 1|\omega_j) = \omega_j, \omega_j \sim Beta(a_\omega, b_\omega), \tag{4}$$

where a_ω and b_ω denote hyperparameters of the Beta distribution. This system yields well-defined posterior conditional distributions for all model parameters, which can be used to perform Gibbs sampling.[4] Averaging over sampled models and risk prices then yields the most likely SDF given the data (Bryzgalova, Huang, & Julliard, 2023).

We utilize Gibbs sampling to compute posterior means and intervals for all unknown parameters and quantities of interest. We focus on the GLS formulations when performing the Bayesian estimations, let the Markov chain run for 500,000 steps in each setting and

[4]For the sake of brevity, we refer to Appendix A of Dickerson et al. (2023) for the system of conditional distributions and a detailed guide for the applied Gibbs sampling.

calculate results after dropping the first 50,000 steps. Further, non-informative prior beliefs are employed about factor inclusion, drawing factor inclusion probabilities from a $Beta(1,1)$ distribution, i.e., setting $a_\omega = b_\omega = 1$ in Equation (4). This results in ex-ante model probabilities of $\frac{1}{2}^{51} \sim 4.44 \times 10^{-16}$. Further, we show our results for multiple levels of shrinkage ψ. We induce these levels by initiating the BMA-SDF estimation for different prior annualized Sharpe ratios ranging from 10% to 90% of the ex-post maximum Sharpe ratio achievable with the set of in-sample test assets.

3. Data

Our main data set for option prices and characteristics is the OptionMetrics Ivy database which hosts historical option prices for U.S. single-name equity options. We obtain option data from OptionMetrics for the period from January 1996 to December 2021. Historical prices, returns, and characteristics of the underlying stocks are sourced from Jensen, Kelly, & Pedersen (2023), a publicly available dataset.[5] We retain only underlyings that are common stocks trading at the NYSE, AMEX, and NASDAQ stock exchanges. Additionally, we exclude stock-month observations if the underlying stock's price is below USD 5. The underlying stock price data is from CRSP. We match CRSP with OpionMetrics using the linking algorithm provided by WRDS. We take daily risk-free rates from Kenneth French's online data library.[6] Monthly risk-free rates are taken from OptionMetrics.

[5]The data, replication code, and documentation can be found at `https://github.com/bkelly-lab/ReplicationCrisis/tree/master/GlobalFactors`.

[6]`https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html`.

We focus on at-the-money options with less than two months to expiration as most of the academic literature studies these contracts due to their high trading volume (see, e.g., Zhan et al., 2022; Vasquez & Xiao, 2023). For each underlying in each month, we select a single call option and a single put option that are closest to at-the-money and that have the shortest maturity among options with more than one month until expiration. Furthermore, we require that the strike-to-spot ratio, K/S, is between 0.8 and 1.2. We restrict our sample to options with a standard expiration on the third Friday of a month. As it is common practice in the literature (see, e.g., Zhan et al., 2022; Bali, Beckmeyer, Moerke, & Weigert, 2023), we apply several filters to construct our option sample.[7] First, we discard options without an implied volatility estimate in the OptionMetrics database. Second, we exclude options on stocks with a dividend payment throughout the month-end to month-end investment period. Third, we drop options for which the bid price is zero, the ask is smaller or equal to the bid, the mid price is below USD 0.125, or the proportional bid-ask-spread is above 50%. Fourth, we exclude observations that violate American option bounds. Fifth, we disregard options with zero open interest over the previous week. Sixth, as most options at the end of each month have the same maturity, we discard observations with different expiration dates from the majority of all other options selected on that day. Finally, we keep only stocks in the sample that have at least one call and one put option available after filtering.

[7]To avoid any forward-looking bias (Duarte, Jones, Mo, & Khorram, 2023), we follow Bali et al. (2023) and apply filters only at position initiation.

3.1. *Option returns*

Our primary units of analysis are monthly delta-hedged option returns. To calculate delta-hedged returns, we first compute delta-hedged option gains following Bakshi & Kapadia (2003). Let $T = \{t = t_0 < \cdots < t_N = t + \tau\}$ denote the partition of the interval from t to $t + \tau$. The delta-hedged option gain is the value of a self-financing portfolio consisting of a long option contract, hedged by a position in the underlying stock such that the sensitivity of the entire option and stock portfolio with respect to changes in the underlying stock price is locally zero. Following Bali et al. (2023), we choose a daily delta-hedging schedule. Tian & Wu (2023) document that delta-hedging at position initiation removes approximately 70% of the directional risks embedded in the option position, whereas daily delta-hedging yields a reduction of 90%. We model long option positions which are hedged discretely N times at each of the dates $t_n, n = 0, \ldots, N-1$. Therefore, the discrete delta-hedged option gain over the period $[t, t+\tau]$ is given by

$$\begin{aligned} \Pi(t, t+\tau) = O_{t+\tau} - O_t - \sum_{n=0}^{N-1} \Delta_{O,t_n} \left[S(t_{n+1}) - S(t_n) \right] \\ - \sum_{n=0}^{N-1} \frac{a_n r_n}{365} \left[O(t_n) - \Delta_{O,t_n} S(t_n) \right], \end{aligned} \tag{5}$$

where O_t denotes the option's mid price at time t, r_n is the risk-free rate at t_n, a_n is the number of calendar days between rehedging dates t_n and t_{n+1} and is set equal to 1, and Δ_{O,t_n} is the observed delta of the option provided by OptionMetrics. Following Cao & Han (2013), we consider gains over an investment horizon of one calendar month to compute month-end

Table 1: Summary statistics of option data.
This table reports summary statistics of our final sample of monthly option data used to construct option factors. The sample is from February 1996 to December 2021. Panel A describes the sample of call options, and Panel B the sample of put options. Daily delta-hedged option returns are the monthly returns of delta-hedged positions. The hedges are adjusted daily to be immune to changes in the underlying. Details are outlined in section 3.1. Open interest is the option contract's open interest at the beginning of the month. Delta is the option's delta as provided by OptionMetrics. Moneyness is the ratio of the option's strike price (K) over the underlying stock price (S). Time to maturity is the days until the option's expiration. Market capitalization is the underlying stock's total market capitalization at the beginning of the month.

Variable	Mean	SD	10^{th} pct.	Median	90^{th} pct.
Panel A: Calls					
Option return (daily delta-hedged, in %)	-0.19	4.56	-4.59	-0.55	4.49
Dollar open interest	1809.81	9176.69	8.55	155	3329.22
Delta	0.54	0.12	0.39	0.54	0.68
Moneyness (K/S)	1	0.05	0.94	1	1.06
Time to maturity (in days)	49.7	2.07	46	50	52
Market capitalization	9.46	37.91	0.34	1.86	17.89
Panel B: Puts					
Option return (daily delta-hedged, in %)	-0.25	3.84	-4.04	-0.61	3.78
Dollar open interest	1280.71	6605.86	7.12	106.25	2289.86
Delta	-0.46	0.12	-0.61	-0.45	-0.31
Moneyness (K/S)	1	0.05	0.94	1	1.06
Time to maturity (in days)	49.69	2.06	46	50	52
Market capitalization	10.47	40.19	0.38	2.11	20.35

to month-end option returns as

$$r_{t,t+\tau} = \frac{\Pi(t, t+\tau)}{|\Delta_t S_t - O_t|}, \tag{6}$$

by dividing the delta-hedged gain, $\Pi(t, t+\tau)$, by the absolute value of the securities involved. Each month, we winsorize the delta-hedged option returns at the 1%-level in both tails to mitigate the impact of erroneous data.

Table 1 provides descriptive statistics of our option return sample for a total of 383,733 call and 339,660 put observations. On average, the monthly option returns with daily delta-hedging schedule are negative with –0.19% for calls and –0.25% for puts, which is in line with a negative volatility risk premium inherent in delta-hedged option positions (Bakshi & Kapadia, 2003). Due to the data selection outlined above, we observe that, on average, the maturity of the option is close to 50 days with a standard deviation of 2 days. Moreover, the average strike-to-spot ratio is 1 with a standard deviation of 0.05, and absolute deltas are close to 0.5.

3.2. Factors

We construct option factors by sorting month t delta-hedged option returns into equal-weighted decile portfolios based on contract-level or stock-level characteristics from month t-1 (sorted from the smallest value to the highest value of the respective characteristic). The option factor returns are given by the $10 - 1$ portfolio return in month t.

3.2.1. Traded factors

As traded factors for the in-sample estimation of BMA posterior probabilities and risk prices, we resort to prominent factors published in the academic literature which have been shown to have explanatory power for the cross-section of (delta-hedged) option returns. Our tradable factor set entails 29 long-minus-short factors constructed with sorts on characteristics such as the difference between implied and realized volatility (`ivrv`; Goyal & Saretto, 2009), idiosyncratic volatility of the underlying stock returns (`ivol`; Cao & Han, 2013), or the average of the ten highest stock returns over the previous quarter (`max10`; Byun & Kim, 2016). Because we also construct an option momentum factor (`omom`) in the spirit of Heston et al. (2023) and Käfer et al. (2023) by sorting on previous option returns from month $t-2$ to $t-12$, the final sample period of our factor sample is from February 1997 to December 2021. Finally, we augment the list of factors by a proxy for the single-name option market return, which we construct following Horenstein et al. (2022). To do so, we use the decile portfolios for each of the 29 characteristics and compute an equal-weighted return across these 290 decile portfolios. Internet Appendix IA1.1.1 describes the characteristics used to construct option factors in detail. In addition, Table 2 provides an overview of all tradable factors as well as their monthly mean returns. In line with the option factor literature, most high-minus-low option factors (23 call and 28 put factors out of 30) yield statistically significant mean returns.

Next to option factors, we also consider factors that are based on stock returns. The addition of stock factors is motivated by Dickerson et al. (2023) who assess the joint pricing power of bond *and* equity factors for corporate bonds using Bayesian model averaging. Moreover, several previous studies in the empirical option return literature use prominent equity factors to explain option return anomalies (e.g., Zhan et al., 2022; Boulatov et al., 2022). For our analyses, we focus on the most widely established equity factors, namely the stock market excess return (`Mkt-Rf`; Sharpe, 1964; Lintner, 1965), the size and value factors (`SMB` and `HML`; Fama & French, 1992), the profitability and investment factors (`RMW` and `CMA`; Fama & French, 2015), and the stock momentum factor (`Mom`; Carhart, 1997). The returns of these stock factors are taken from Kenneth French's online data library.

Table 2: Overview of factor set (traded option factors).
This table lists the 30 option factors used in our tradeable factor set with the original paper that proposed the factor or documented the characteristics' explanatory power for the cross-section of option returns. On the right hand, we report monthly means of factor returns for factors constructed using delta-hedged call or put returns. t-stats of mean returns account for heteroskedasticity and autocorrelation in residuals up to lag four, following Newey & West (1987). The sample period is from February 1997 to December 2021. Detailed descriptions of the characteristics used for factor construction are documented in Internet Appendix IA1.1.1.

		Monthly mean returns in % (t-stat)	
Factor	Reference paper	Calls	Puts
Embedded leverage (`embedlev`)	Frazzini & Pedersen (2022)	0.45 (7.98)	0.52 (9.92)
Delta-hedging costs (`hc`)	Tian & Wu (2023)	-0.87 (-7.08)	-0.82 (-9.83)
Volatility risk (`vr`)	Tian & Wu (2023)	-1.06 (-9.00)	-0.98 (-11.99)
Historical jump risk (`jr`)	Tian & Wu (2023)	-0.83 (-12.82)	-0.66 (-14.31)
Volatility of implied volatility (`vov`)	Ruan (2020)	-0.39 (-5.25)	-0.38 (-6.72)
Option illiquidity (`optspread`)	Christoffersen et al. (2018)	0.001 (0.018)	0.07 (1.16)
Option momentum (`omom`)	Heston et al. (2023); Käfer et al. (2023)	1.16 (11.83)	1.00 (13.56)
Historical stock volatility (`hvol`)	Hu & Jacobs (2020)	-0.70 (-4.70)	-0.77 (-6.76)
Systematic volatility (`sysvol`)	Aretz, Lin, & Poon (2023)	-0.08 (-0.55)	-0.13 (-1.19)
Impl. volatility term structure (`ivterm`)	Aretz et al. (2023)	-0.91 (-7.88)	-0.78 (-9.11)
Stock return autocorrelation (`ac`)	Jeon, Kan, & Li (2019)	0.022 (0.39)	-0.11 (-2.52)
Average of 10 highest past returns (`max10`)	Bali, Cakici, & Whitelaw (2011)	-0.58 (-3.88)	-0.67 (-5.74)
Default risk (`defrisk`)	Vasquez & Xiao (2023)	-0.22 (-1.54)	-0.32 (-3.25)
Idiosyncratic skewness (`iskew`)	Byun & Kim (2016)	-0.06 (-1.18)	-0.13 (-2.78)
Total skewness (`tskew`)	Byun & Kim (2016)	-0.11 (-1.61)	-0.18 (-3.54)
Idiosyncratic volatility (`ivol`)	Cao & Han (2013)	-0.84 (-6.58)	-0.90 (-9.02)
Implied minus realized volatility (`ivrv`)	Goyal & Saretto (2009)	-2.34 (-10.76)	-1.84 (-11.82)
Stock illiquidity (`amihud`)	Kanne et al. (2023); Zhan et al. (2022)	-0.58 (-4.76)	-0.58 (-7.02)
Short interest (`rsi`)	Ramachandran & Tayal (2021)	-0.21 (-2.96)	-0.53 (-9.54)
1-year new stock issues (`issue_1y`)	Zhan et al. (2022)	-0.43 (-4.35)	-0.39 (-4.49)
5-year new stock issues (`issue_5y`)	Zhan et al. (2022)	-0.58 (-5.95)	-0.47 (-6.59)
Analyst dispersion (`disp`)	Zhan et al. (2022)	-0.35 (-4.64)	-0.33 (-6.11)
Altman Z-score (`zscore`)	Zhan et al. (2022)	0.23 (2.44)	0.18 (2.54)
Cash-to-assets ratio (`cash_at`)	Zhan et al. (2022)	-0.86 (-7.87)	-0.70 (-8.12)
Cash flow volatility (`ocfq_saleq_std`)	Zhan et al. (2022)	-0.91 (-10.21)	-0.76 (-11.88)
Operating profits to book equity (`ope_be`)	Zhan et al. (2022)	0.84 (8.59)	0.75 (10.37)
Profit margin (`ebit_sale`)	Zhan et al. (2022)	0.94 (9.39)	0.83 (11.66)
Net total issuance (`netis_at`)	Zhan et al. (2022)	-0.53 (-4.84)	-0.50 (-6.26)
Stock price (`log_price`)	Zhan et al. (2022); Boulatov et al. (2022)	1.02 (7.26)	0.80 (8.01)
Option market factor (`ew_ret`)	Horenstein et al. (2022)	-0.16 (-1.31)	-0.24 (-2.36)

3.2.2. Non-traded factors

We supplement the tradeable factors described in the previous subsection with 15 non-traded factors. First, we use ten of the non-traded factors in Bryzgalova, Huang, & Julliard (2023) and Dickerson et al. (2023) that might span the risks affecting option prices, such as the economic uncertainty risk (Jurado, Ludvigson, & Ng, 2015) or aggregate volatility risk as captured by first differences in the CBOE VIX index (Ang, Hodrick, Xing, & Zhang, 2006). Second, we supplement these factors with five additional non-traded factors that proxy for risks relevant for explaining returns in the options market. For instance, we use payoffs of S&P 500 correlation swaps to proxy for correlation risk as Driessen, Maenhout, & Vilkov (2009) show that correlation risk exposure can explain the cross-section of single-name option returns. Further details on the definition of the non-traded factors are reportet in Internet Appendix IA1.2.

3.3. Test assets

We require a set of in-sample and out-of-sample test assets to implement the BMA methodology outlined in Section 2. The BMA approach uses the in-sample test assets to determine posterior factor inclusion probabilities and posterior mean factor risk premia. These risk prices then serve as input for cross-sectional out-of-sample tests in which we evaluate the pricing performance of the BMA-SDF and other benchmark models which are described in Internet Appendix IA2.

3.3.1. In-sample test assets

We include the 30 traded *option* factors described in Section 3.2.1 in our set of in-sample test assets.[8] As stressed by Barillas & Shanken (2017), the inclusion of the traded option

[8]Note that while stock factors are tradable, we do not include them as test assets, because pricing stock factors is not a concern of this study. Therefore, the stock factors are treated as non-traded factors and are part of $f_t^{(2)} \in \mathbb{R}^{K_2}$ in the BMA-SDF estimation.

factors ensures that factors included in a model can also price excluded candidate factors and themselves (Bryzgalova, Huang, & Julliard, 2023). Crucially, we do not include the traded *stock* factors in the set of in-sample test assets as we are not interested in the option BMA-SDF pricing stock-level risk factors. Furthermore, we include 5×5 independently, double-sorted option portfolios in the spirit of the Fama-French portfolios sorted on size and the book-to-market ratio. These portfolios are established test assets in the stock pricing literature. As an analogous option size characteristic, we sort on the option's outstanding dollar-open interest in month $t-1$. As an option value characteristic, we use implied minus realized volatility (`ivrv`) of the option contract in $t-1$ as defined in Goyal & Saretto (2009). `ivrv` is similar to the book-to-market ratio of stocks because the Black-Scholes (1973) implied volatility can be interpreted as a measure of market value and realized volatility as a measure of fundamental option value (Karakaya, 2014). In total, our set of in-sample test assets consists of 30 long-short and 25 long-only option portfolios.

3.3.2. Out-of-sample test assets

For our set of cross-sectional out-of-sample test assets, we follow Dickerson et al. (2023) in sorting option returns into monthly *long* portfolios based on the 17 Fama-French industry classification (FF17). Sorting option returns by industries results in long portfolios with sufficient return variation in the cross-section. Moreover, we consider additional long-short option portfolios that are *not* included in our set of candidate option factors for the out-of-sample tests. To do so, we turn to the option and stock-level characteristics in Goyal & Saretto (2024) that are not sorting characteristics for the traded factors in Section 3.2.1.

We again construct monthly factors with a decile sort and define factor returns as the high-minus-low decile return spread. The procedure results in 26 additional return anomalies, such as factors based on the option contract's mid price or the underlying stock's book-to-market ratio. Details on the definition of the Goyal & Saretto (2024) option anomalies are in Internet Appendix IA1.4.2. Overall, our set of cross-sectional out-of-sample test assets consists of 43 option portfolios.

4. Empirical results

We split our baseline analysis into five parts. To answer which factors are most important to price the cross-section of our test assets, we first highlight the factors for which the BMA yields the highest posterior factor probabilities. Second, we assess the model probability and dimensionality of the estimated BMA-SDF. In the third part, we compare the pricing performance of the BMA-SDF estimations to previously proposed low-dimensional factor models, both in-sample and out-of-sample for a different *cross-section* of test assets. Next, we analyze the out-of-sample performance of a reduced-form linear option factor model that includes factors based on BMA-implied posterior probabilities. In the fifth part, we assess the out-of-sample performance for a different *time-series* of the same test assets used to estimate the BMA-SDF.

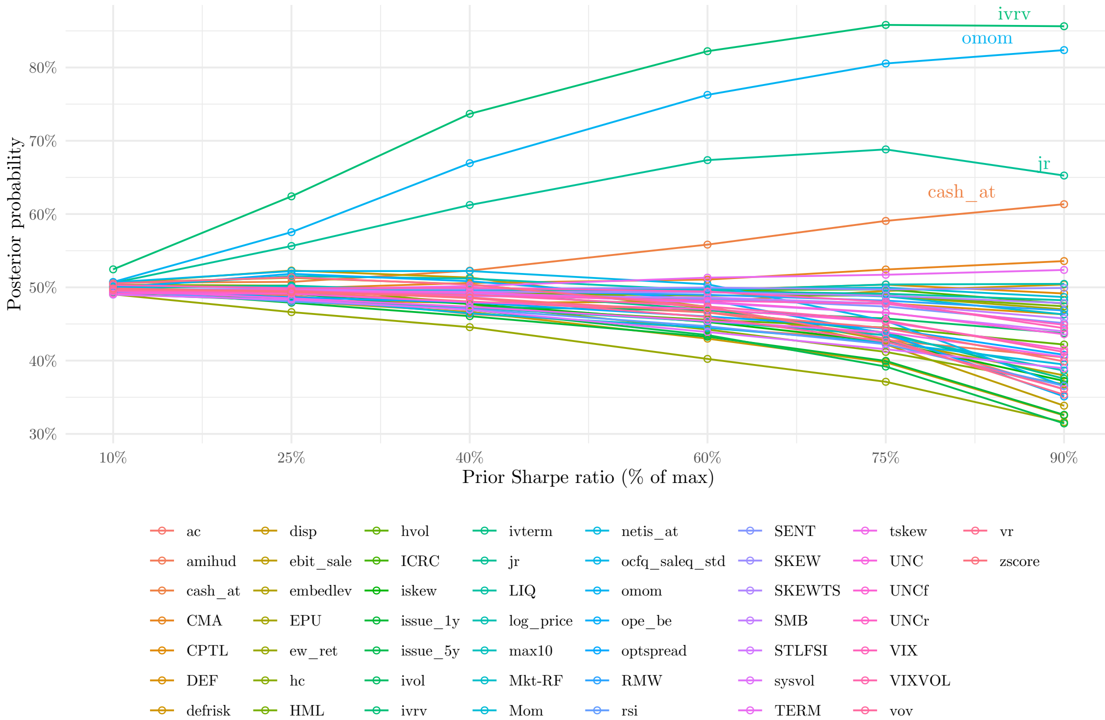


Fig. 1. Posterior factor inclusion probabilities - Calls

This figure shows posterior factor probabilities $\mathbb{E}[\gamma_j|\text{data}]$ estimated with the BMA approach outlined in Section 2. The factor set includes returns of 30 traded long-short factors based on delta-hedged call returns as well as 21 non-traded factors from February 1997 to December 2021. Additional test assets are 5×5 long portfolios based on independent monthly sorts on `ivrv` and `doi`. Portfolio returns are calculated with equal option weighting. We use non-informative flat priors on factor inclusion probability drawn from a $Beta(1, 1)$ distribution and different prior annualized Sharpe ratios ranging from 10% to 90% of the ex-post maximum achievable Sharpe ratio.

4.1. Posterior factor inclusion probabilities and risk prices

We report posterior factor inclusion probabilities, $\mathbb{E}[\gamma_j|\text{data}]$, and posterior prices of risk, $\mathbb{E}[\lambda_j|\text{data}]$, for each factor j and different prior beliefs about the maximum Sharpe ratio. Posterior probabilities as a function of prior Sharpe ratios are shown in Figure 1 for factors based on calls and in Figure 2 for factors based on puts. Tables A1 and A2 present posterior probabilities and risk prices in tabular form.

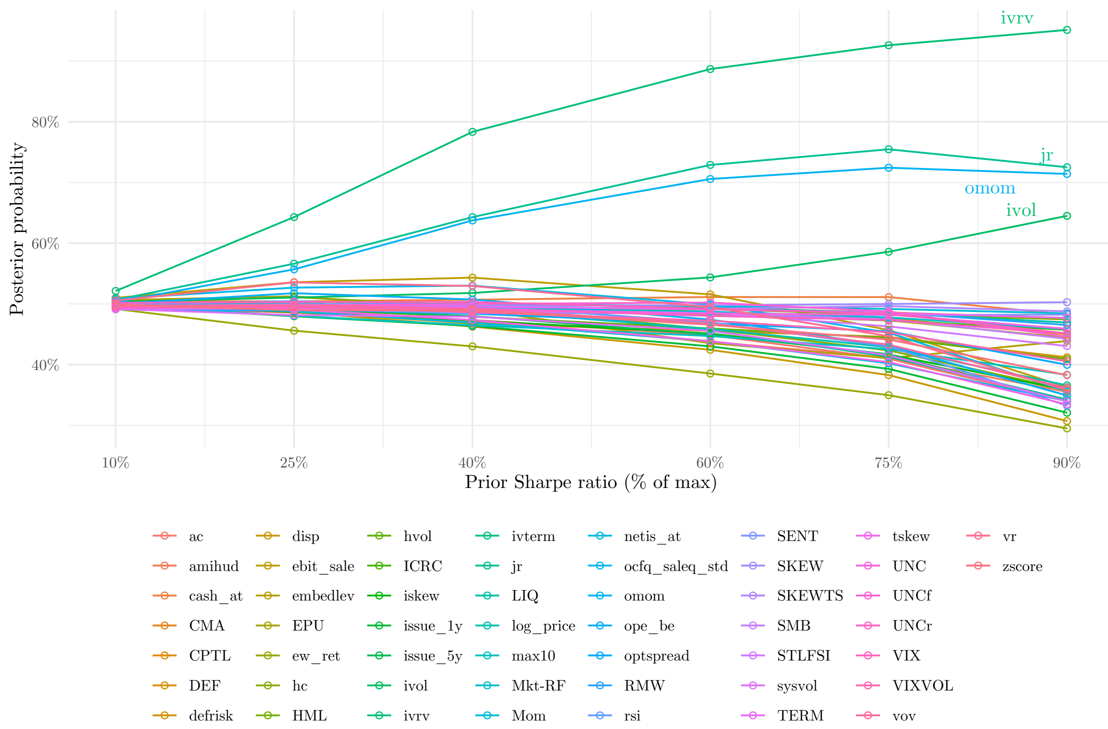


Fig. 2. Posterior factor inclusion probabilities - Puts

This figure shows posterior factor probabilities $\mathbb{E}[\gamma_j|\text{data}]$ estimated with the BMA approach outlined in Section 2. The factor set includes returns of 30 traded long-short factors based on delta-hedged put returns as well as 21 non-traded factors from February 1997 to December 2021. Additional test assets are 5×5 long portfolios based on independent monthly sorts on `ivrv` and `doi`. Portfolio returns are calculated with equal option weighting. We use non-informative flat priors on factor inclusion probability drawn from a $Beta(1, 1)$ distribution and different prior annualized Sharpe ratios ranging from 10% to 90% of the ex-post maximum achievable Sharpe ratio.

For delta-hedged calls, even under high levels of shrinkage, three factors stand out by exhibiting posterior inclusion probability higher than the prior probability of 50% and are thus likely to be included in the true SDF. These factors are `ivrv`, `omom`, and `jr`. `ivrv` and `jr` show negative posterior risk prices under all prior maximum Sharpe ratios, which is consistent with the findings of Goyal & Saretto (2009) and Tian & Wu (2023). After accounting for all other risks, `ivrv` might capture residual mispricing in options.[9] A high spread in implied

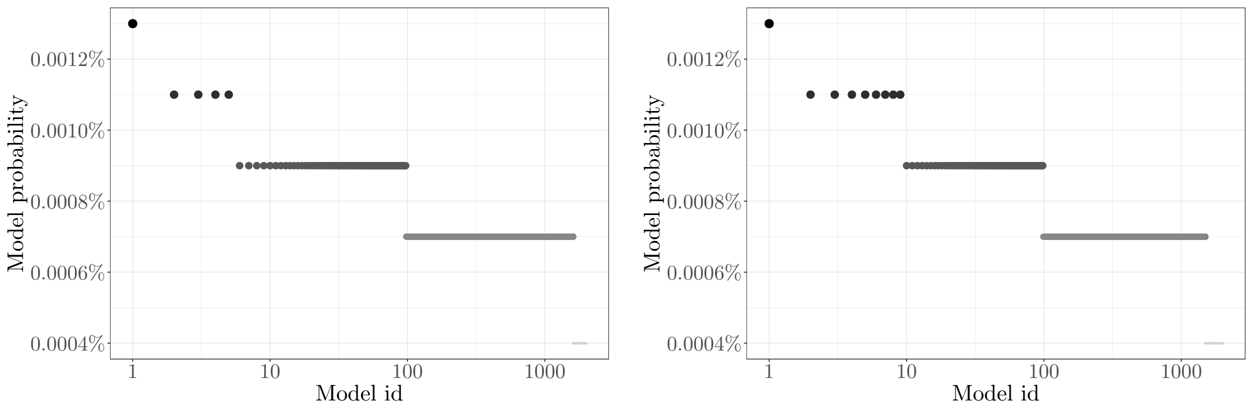
[9] As `ivrv` together with `doi` is used for the double-sort to construct the 25 long-only test assets, the high importance of the factor might be mechanical. However, we re-estimate the BMA using instead a double sort on `be_me` and `mcap` to construct the 25 long-only test assets. As reported in Internet Appendix IA3,

versus realized volatility on average indicates overpricing, and thus, such options earn lower consequent returns. On the other hand, `jr` captures the risks of market makers that demand a premium for options with high jump risk. The positive posterior risk price for `omom` is consistent with the findings of Heston et al. (2023) and Käfer et al. (2023), who find that past performance predicts future performance in delta-neutral option positions. In this context, Tian & Wu (2023) point out that option momentum effects might be driven by persistence in the risk magnitude variations of the underlying, unspecified risk sources.[10] As in Bryzgalova, Huang, & Julliard (2023), a large number of factors is assigned a posterior probability very close to the prior probability, indicating that these factors are weakly identified. However, for less shrinkage, most of these factors become unlikely candidates for the SDF. Interestingly, a group of correlated factors including `ebit_sale`, `ope_be`, `ocfq_saleq_std`, and `cash_at` are moderately likely candidates of priced risk for low model complexity, but when less shrinkage is imposed only `cash_at` is selected more times than expected. This finding points to a more pronounced role of model selection over model aggregation when regularization is limited. Also noteworthy is that under moderate to high shrinkage, no non-traded factor reaches posterior inclusion probabilities larger than 50.6%.

For delta-hedged puts, a similar picture arises. Again, the most important factors are `ivrv`, `omom`, and `jr` with posterior inclusion probability significantly larger than 50% across all prior maximum Sharpe ratios. The signs of risk prices remain consistent with expectations

`ivrv` remains the factor with the highest posterior inclusion probability.

[10]One might argue that `ivrv` and `omom` might capture multiple sources of risks linked to more interpretable characteristics. However, in Internet Appendix IA4, we replicate the analysis without those two factors. No factor with previously low inclusion probability steps up to take the place of `ivrv` and `omom`, indicating that they are not driving out other factors.



(a) Calls **(b) Puts**

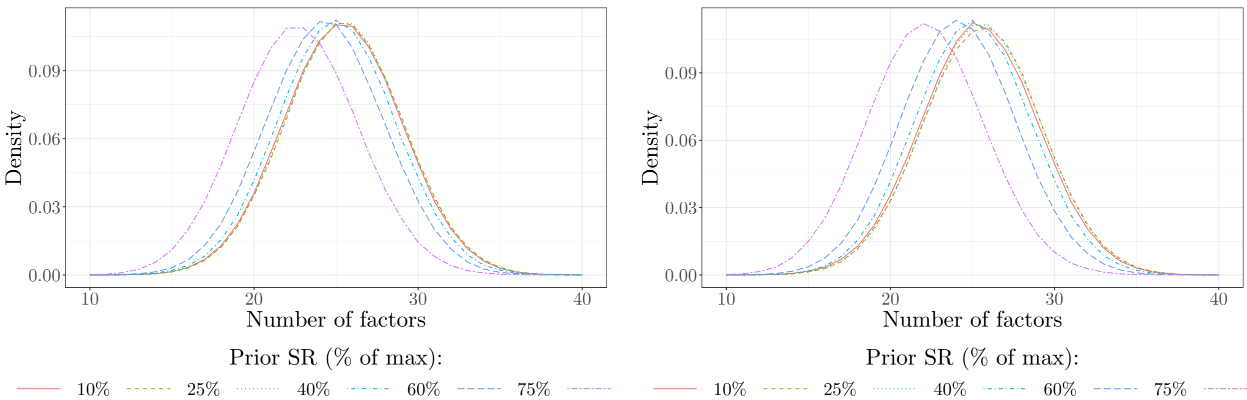
Fig. 3. Model probability

This figure shows model probabilities of the most common 2000 models observed in the final 450,000 Markov chain elements of the BMA-SDF estimation with a prior Sharpe ratio of 75% of the ex-post maximum Sharpe ratio. The left panel (a) is based on calls, and the right panel (b) is based on puts. All other specifications of the BMA follow those detailed in Figure 1.

that arise from prior literature. Interestingly, the posterior inclusion probability for `jr` is slightly higher for puts than for calls, providing evidence that jump risk in the options' underlying is even more influential in the pricing of puts than in the pricing of calls. Instead of `cash_at`, `ivol` wins the horse race as the fourth important factor in pricing the put test assets for low regularization.

4.2. In-sample model uncertainty and dimensionality

Next, we discuss the model probability and model dimensionality of our in-sample BMA-SDF estimation. In Figure 3, we plot the probability of the 2,000 most common factor combinations in the final 450,000 Markov chain elements with a prior maximum Sharpe ratio of 75% of the ex-post maximum Sharpe ratio. Panel (a) shows the BMA-SDF estimation for calls, and Panel (b) shows the BMA-SDF estimation for puts, respectively. Very similar



(a) Calls   **(b) Puts**

Fig. 4. Number of factors

This figure displays the distribution of the number of factors in models chosen by the final 450,000 Markov chain elements of the BMA-SDF estimation for different prior Sharpe ratios. The left panel (a) is based on calls and the right panel (b) is based on puts. All other specifications of the BMA follow those detailed in Figure 1.

to the results of Bryzgalova, Huang, & Julliard (2023) for the cross-section of stock return, there is no stand-out model. For calls, the most common model arises six times, followed by four more models arising five times. For puts, again only one model arises six times, resulting in a probability of less than a basis point. Further, there are close to one hundred models that appear four times. While these posterior model probabilities are significantly higher than the ex-ante probability of $\frac{1}{2}^{51} \sim 4.44 \times 10^{-16}$, they indicate that there are no hugely dominant factor combinations but rather large sets of similarly probable models. We count the number of factors in each of the final 450,000 Markov chain elements for different prior maximum Sharpe ratios and plot the distribution of model dimensionality. For high shrinkage, this distribution centers around the expected number of 0.5×51 factors, namely the prior inclusion probability of each factor times the number of factors.

Evidently, the data cannot sufficiently rule out that the true SDF is rather sparse, giving more weight to dense models and high dimensionality. Only for the largest prior Sharpe ratio does the average number of factors drop significantly. As previously described and explained by Bryzgalova, Huang, & Julliard (2023), this result is likely due to rather weakly identified factors driving out other factors when less shrinkage is applied. In that case, the BMA leads to moderately more model selection rather than aggregation.

4.3. In-sample and cross-sectional out-of-sample asset pricing

We benchmark the BMA-SDF to previously proposed low-dimensional factor models with regard to their cross-sectional pricing power. The benchmark models include the three-factor model of Horenstein et al. (2022), the two-factor model of Zhan et al. (2022), the five-factor model of Tian & Wu (2023), and the four-factor model of Agarwal & Naik (2004). The models are described in detail in Internet Appendix IA2. Additionally, we add a model including all 51 factors as well as an option version of the single-factor CAPM, where we use `ew_ret` as the sole factor. For all benchmark models, we use GMM with a GLS weighting matrix to estimate risk prices. For the BMA-SDF, we use the posterior risk prices estimated with various prior maximum Sharpe ratios reported in Table A1 and Table A2. An intercept is added to all models.

We calculate average in-sample pricing errors for the 30 traded factors and the 25 portfolios sorted on `ivrv` and `doi`. All returns are standardized to an annual volatility of 100%. Following Bryzgalova, Huang, & Julliard (2023), we report the root mean square error (RMSE), the mean absolute percentage error (MAPE), as well as R-squared values both without (R^2_{ols})

Table 3: Cross-sectional pricing performance - Calls.

This table reports four performance measures of cross-sectional pricing for different factor models. For the BMA-SDF, prices of risk are estimated using the methodology outlined in Section 2 and reported in Table A1. For the benchmark models, we use GMM with a GLS weighting matrix to estimate risk prices. Benchmark models are described in Internet Appendix IA2. *CAPM* refers to a one-factor model utilizing only `ew_ret`, whereas *51 factors* utilizes all 30 traded and 21 non-traded factors. Out-of-sample test assets are the 26 long-short factors detailed in Internet Appendix IA1.4.2 as well as 17 long portfolios based on FF17 industry sorts. Portfolio returns are calculated with equal call option weighting. RMSE and MAPE are based on returns standardized to an annual volatility of 100%.

Panel A: In-Sample Pricing, Test Assets: 51 Factors and 25 IVRV-DOI portfolios									
BMA-SDF	RMSE	MAPE	R^2_{ols}	R^2_{gls}	Model	RMSE	MAPE	R^2_{ols}	R^2_{gls}
10%-SR_{pr}	1.348	1.03	0.115	0.059	51 factors	0.078	0.042	0.997	0.934
25%-SR_{pr}	1.099	0.846	0.387	0.162	CAPM	1.421	1.104	0.008	0.028
40%-SR_{pr}	0.884	0.681	0.604	0.278	HVX	0.971	0.686	0.536	0.265
60%-SR_{pr}	0.632	0.484	0.798	0.419	ZHCT	1.217	0.963	0.307	0.077
75%-SR_{pr}	0.477	0.369	0.885	0.512	AN	1.225	1.031	0.237	0.13
90%-SR_{pr}	0.347	0.279	0.939	0.605	TW	1.408	1.172	0.353	0.364
Panel B: Out-of-Sample Pricing, Test Assets: 26 Factors and 17 Industry Portfolios									
BMA-SDF	RMSE	MAPE	R^2_{ols}	R^2_{gls}	Model	RMSE	MAPE	R^2_{ols}	R^2_{gls}
10%-SR_{pr}	1.407	0.934	0.151	0.009	51 factors	1.534	1.113	-0.007	-3.914
25%-SR_{pr}	1.13	0.774	0.453	0.145	CAPM	1.491	1.012	0.048	-0.011
40%-SR_{pr}	0.94	0.638	0.622	0.243	HVX	1.029	0.863	0.546	0.298
60%-SR_{pr}	0.769	0.5	0.746	0.336	ZHCT	1.078	0.825	0.502	0.044
75%-SR_{pr}	0.682	0.427	0.8	0.383	AN	1.249	1.007	0.332	0.057
90%-SR_{pr}	0.619	0.382	0.836	0.418	TW	1.329	1.004	0.244	-0.196

and with a weighting matrix (R^2_{gls}). For out-of-sample tests, we utilize the same estimated risk prices but use them to price 26 long-short factors detailed in Internet Appendix IA1.4.2 and proposed by Goyal & Saretto (2024) as well as 17 long portfolios based on FF17 industry sorts. Pricing performance is reported in Table 3 for assets constructed from calls and in Table 4 for assets constructed from puts.

Several notable observations can be drawn. First, pricing performance improves the less shrinkage is applied. This is true for both puts and calls and all four performance metrics. Even for out-of-sample pricing, it is beneficial to use a high prior maximum Sharpe ratio. There seems to be no problem with overfitting in the BMA approach even when allowing high

Table 4: Cross-sectional pricing performance - Puts.
This table reports four performance measures of cross-sectional pricing for different factor models. For the BMA-SDF, prices of risk are estimated using the methodology outlined in Section 2 and reported in Table A2. For the benchmark models, we use GMM with a GLS weighting matrix to estimate risk prices. Benchmark models are described in Internet Appendix IA2. *CAPM* refers to a one-factor model utilizing only `ew_ret`, whereas *51 factors* utilizes all 30 traded and 21 non-traded factors. Out-of-sample test assets are the 26 long-short factors detailed in Internet Appendix IA1.4.2 as well as 17 long portfolios based on FF17 industry sorts. Portfolio returns are calculated with equal put option weighting. RMSE and MAPE are based on returns standardized to an annual volatility of 100%.

Panel A: In-Sample Pricing, Test Assets: 51 Factors and 25 IVRV-DOI portfolios									
BMA-SDF	RMSE	MAPE	R^2_{ols}	R^2_{gls}	Model	RMSE	MAPE	R^2_{ols}	R^2_{gls}
10%-SR_{pr}	1.518	1.175	0.116	0.047	51 factors	0.035	0.02	0.999	0.988
25%-SR_{pr}	1.203	0.921	0.395	0.17	CAPM	1.568	1.247	-0.008	0.011
40%-SR_{pr}	0.949	0.713	0.614	0.301	HVX	1.346	0.97	0.268	0.252
60%-SR_{pr}	0.67	0.501	0.806	0.451	ZHCT	1.229	0.919	0.362	0.108
75%-SR_{pr}	0.503	0.384	0.89	0.54	AN	1.508	1.153	0.104	0.072
90%-SR_{pr}	0.369	0.297	0.941	0.619	TW	1.36	1.081	0.444	0.376
Panel B: Out-of-Sample Pricing, Test Assets: 26 Factors and 17 Industry Portfolios									
BMA-SDF	RMSE	MAPE	R^2_{ols}	R^2_{gls}	Model	RMSE	MAPE	R^2_{ols}	R^2_{gls}
10%-SR_{pr}	1.325	0.92	0.169	-0.265	51 factors	1.532	1.192	-0.111	-3.616
25%-SR_{pr}	1.017	0.741	0.511	0.037	CAPM	1.403	1	0.069	-0.138
40%-SR_{pr}	0.802	0.607	0.696	0.221	HVX	1.371	1.113	0.111	0.234
60%-SR_{pr}	0.607	0.461	0.825	0.377	ZHCT	0.885	0.733	0.629	0.118
75%-SR_{pr}	0.505	0.386	0.88	0.459	AN	1.41	1.012	0.059	-0.065
90%-SR_{pr}	0.431	0.332	0.912	0.525	TW	1.227	0.934	0.289	-0.111

model complexity. Second, and in contrast to the lack of overfitting when using the BMA-SDF, the 51-factor model exhibits the highest in-sample pricing performance but performs badly in the out-of-sample tests. On the other hand, the BMA-SDF estimations beat all other benchmark models, both in-sample and out-of-sample, when the prior Sharpe ratio is higher or equal to 60% of the ex-post maximum Sharpe ratio. Again, this observation holds for all four performance metrics and for both calls and puts. For the highest considered prior Sharpe ratio, and therefore the lowest level of shrinkage, the BMA-SDF achieves an extraordinarily high out-of-sample R^2_{ols} of 0.84 for calls and 0.91 for puts. The benchmark models perform significantly worse, even though some of them include the factors that were assigned the highest posterior inclusion probability. As am example. the factor model of Tian & Wu (2023) includes three of the four stand-out factors, namely `ivrv`, `jr`, and `omom`, but yields worse in-sample and out-of-sample results. This weaker performance can arise from both different risk price estimations for these factors in the GMM versus the BMA, or from other factors in the BMA-SDF that yield additional explanatory power in the cross-section of call and option prices. In Internet Appendix IA5, we show the out-of-sample pricing performance measures separately for the industry portfolios and the 26 long-short portfolios and also add a third set of test assets, namely 25 long portfolios based on independent 5 $\times$ 5 sorts on the book-to-market ratio (`be_me`) and the market capitalization (`mcap`) of the options' underlying stocks. Again, the BMA-SDF significantly reduces pricing errors for all three sets of test assets compared to the benchmark models.

4.4. Pricing performance of reduced-form models implied by the BMA-SDF

So far, we have reported significant improvements in the asset pricing performance of the BMA-SDF over previously proposed low-dimensional benchmark models. Together with the large average number of factors in the models proposed by the BMA, this provides evidence that the true SDF is rather dense than sparse. Nonetheless, in this section, we test whether the worse performance of the benchmark models is due to a lack of model dimensionality or a weaker selection of factors. To do so, we introduce new "best factor" models. Factors with the highest posterior inclusion probabilities, as yielded by this BMA-SDF estimation process with a prior maximum Sharpe ratio of 60% of the ex-post maximum Shape ratio,

Table 5: Pricing performance of factors with highest posterior inclusion probability.

This table reports four out-of-sample performance measures of cross-sectional pricing for different factor models, based both on calls and puts. For the first 4 models, factors with the highest ex-post inclusion probabilities are included. These probabilities are taken from the BMA estimation using a prior maximum Sharpe ratio of 75% of the ex-post maximum Sharpe ratio. Benchmark models are described in Internet Appendix IA2. For all models, we use GMM with a GLS weighting matrix to estimate risk prices in-sample. Out-of-sample test assets are the 26 long-short factors detailed in Internet Appendix IA1.4.2 as well as 17 long portfolios based on FF17 industry sorts. Portfolio returns are calculated with equal option weighting. RMSE and MAPE are based on returns standardized to an annual volatility of 100%.

	Panel A: Out-of-Sample Pricing - Calls				Panel B: Out-of-Sample Pricing - Puts			
Models	RMSE	MAPE	R^2_{ols}	R^2_{gls}	RMSE	MAPE	R^2_{ols}	R^2_{gls}
Best factor	1.07	0.844	0.51	0.209	1.415	1.089	0.053	0.103
Best 4 factors	0.705	0.517	0.787	0.025	0.494	0.39	0.884	0.528
Best 10 factors	0.605	0.427	0.843	0.336	0.412	0.338	0.92	0.533
Best 25 factors	0.657	0.429	0.815	0.25	0.425	0.334	0.915	0.4
51 factors	1.534	1.113	-0.007	-3.914	1.532	1.192	-0.111	-3.616
CAPM	1.491	1.012	0.048	-0.011	1.403	1	0.069	-0.138
HVX	1.029	0.863	0.546	0.298	1.371	1.113	0.111	0.234
ZHCT	1.078	0.825	0.502	0.044	0.885	0.733	0.629	0.118
AN	1.249	1.007	0.332	0.057	1.41	1.012	0.059	-0.065
TW	1.155	0.877	0.429	0.009	1.227	0.934	0.289	-0.111

are chosen for these models. In Table 5, we show the out-of-sample pricing performance of four such models with one, four, ten, and 25 factors, respectively. Risk prices are not taken from the BMA-SDF but rather estimated with GMM and a GLS weighting matrix.

Our results indicate that, for both calls and puts, the "best factor" models with more than one factor outperform all of the benchmark models. Only the model of Horenstein et al. (2022) for calls, the model of Zhan et al. (2022) for puts, and the 51-factor model yield better pricing than the single-factor model with `ivrv` as the sole factor. Interestingly, the four-factor models with `ivrv`, `jr`, and `omom` strongly outperform even the model of Tian & Wu (2023) which shares those three factors. Adding `cash_at` as a fourth call factor and `ivol` as a fourth put factor greatly enhances the pricing performance of the models. Overall, our results indicate that based on pricing performance there exists an optimal number of factors in this approach. While the ten-factor model improves upon the performance of the four-factor model, there is a drop in out-of-sample pricing power when including the best 25 factors or even all factors. Nevertheless, we conclude that the BMA approach is useful, even if only for model selection based on the posterior inclusion probabilities, as seemingly relevant factors are selected. Even without the posterior risk prices, we can construct models that beat previously proposed factor models in pricing the cross-section of options.

4.5. Out-of-sample asset pricing using different time periods

Whereas the previous two subsections focus on the out-of-sample performance of the BMA-SDF and several benchmark models using a different *cross-section* of test assets, we now turn to assess the *time-series* pricing performance of the option factor models. To do so, we split the sample into two halves, namely from March 1997 to July 2009 and from August 2009 to December 2021.[11] We then determine posterior factor probabilities and factor risk prices over the first (second) subperiod and evaluate the out-of-sample performance using the second (first) subperiod of factors and in-sample test assets. Similarly, for our reduced-form benchmark models, we first determine risk prices via GMM using the first (second) subperiod and subsequently evaluate the pricing performance by predicting the returns of the test assets in the second (first) subperiod. This sample-split approach is common in the literature for time-series out-of-sample analysis (Linnainmaa & Roberts, 2018; Gu, Kelly, & Xiu, 2020).[12]

[11] As our factor returns cover a total period of 299 months, we omit the first-month factor return observations of February 1997 to obtain two halves of equal length.

[12] In this context, Bryzgalova, Huang, & Julliard (2023) point out that it would be optimal to assess time-series out-of-sample performance over factors' post-publication period. However, the authors also state that

Initially, we again focus on the posterior probabilities resulting from the spike-and-slab approach outlined in Section 2. Figure B1 displays factor probabilities for calls, Figure B2 for puts. In each figure, the upper Panel (a) shows posterior factor probabilities estimated over the first sample subperiod as a function of the prior Sharpe ratios set to a range of 10%, 25%, 40%, 60%, 75%, and 90% of the ex-post maximum Sharpe ratio, Panel (b) includes posterior probabilities for estimations over the second subperiod. Strikingly, we find that for the different estimation periods, the overall composition of option factors that are likely part of the option SDF changes. For calls, during the first sub-estimation period, `ivrv` is the most likely factor to be included in the SDF. Other factors that have a posterior probability of clearly above 50% are `omom` and the stock investment factor `CMA` that gains in importance for smaller degrees of shrinkage. `jr` strongly falls off in terms of posterior probability for smaller prior Sharpe ratios. For the second subperiod, the overall insights from the posterior probabilities change slightly. Now, `ivrv` and `omom` are in a very close race for the most probable factor in the SDF, with `ivrv` only slightly winning out for the smallest imposed degree of shrinkage. Moreover, `jr` is a likely factor candidate included in the SDF.

The differences between estimation periods are also pronounced for puts as can be seen in Figure B2. For the first subperiod, `ivrv` is by a large margin the most likely part of the SDF in the options market. Compared to `ivrv`, `omom` and `jr` only seem to play a minor role comparatively with exterior probabilities of roughly 55% at the prior Sharpe ratios of 60% of the maximum. The results for the posterior probabilities change considerably in the second

due to factors in their sample being only recently featured in published academic work, this ideal approach is not feasible. For our sample of option factors, the relevant publications for most factors tend to be even more recent than for many established stock factors. Hence, we rely on an even sample split for our purpose.

Table 6: Out-of-sample pricing performance for different estimation and evaluation periods - Calls.

This table reports four performance measures of time-series out-of-sample pricing for different call factor models. In Panel A (B), we use the first (second) half of the sample period, March 1997 to June 2006 (July 2006 to December 2021), for the model estimation and recovery of risk prices and evaluate its pricing ability on the omitted half. For the BMA-SDF, prices of risk are estimated using the methodology outlined in Section 2. For the benchmark models, we use GMM with a GLS weighting matrix to estimate risk prices. Benchmark models are described in Internet Appendix IA2. *CAPM* refers to a one-factor model utilizing only `ew_ret`, whereas *51 factors* utilizes all 30 traded and 21 non-traded factors. Out-of-sample test assets are the 26 long-short factors detailed in Internet Appendix IA1.4.2 as well as 17 long portfolios based on FF17 industry sorts. Portfolio returns are calculated with equal call option weighting. RMSE and MAPE are based on returns standardized to an annual volatility of 100%.

Panel A: estimation period: March 1997 to July 2009, evaluation period: August 2009 to December 2021

BMA-SDF	RMSE	MAPE	R^2_{ols}	R^2_{gls}	Model	RMSE	MAPE	R^2_{ols}	R^2_{gls}
10%-SR_{pr}	1.628	1.226	0.101	0.018	51 factors	6.537	5.724	-10.986	-11.164
25%-SR_{pr}	1.337	0.994	0.383	0.067	CAPM	1.753	1.329	-0.024	0.006
40%-SR_{pr}	1.054	0.792	0.63	0.118	HVX	1.979	1.511	-0.454	-0.079
60%-SR_{pr}	0.834	0.698	0.807	0.169	ZHCT	1.946	1.464	-0.262	-0.048
75%-SR_{pr}	0.913	0.799	0.782	0.185	AN	1.826	1.475	-0.319	-0.139
90%-SR_{pr}	1.16	1.025	0.616	0.176	TW	1.914	1.451	-0.386	-0.053

Panel B: estimation period: August 2009 to December 2021, evaluation period: March 1997 to July 2009

BMA-SDF	RMSE	MAPE	R^2_{ols}	R^2_{gls}	Model	RMSE	MAPE	R^2_{ols}	R^2_{gls}
10%-SR_{pr}	1.448	1.141	0.052	-0.005	51 factors	14.549	11.211	-94.935	-37.209
25%-SR_{pr}	1.403	1.129	0.134	0.028	CAPM	1.493	1.185	0.01	-0.031
40%-SR_{pr}	1.37	1.123	0.212	0.065	HVX	1.594	1.298	-0.01	0.006
60%-SR_{pr}	1.241	1.031	0.368	0.121	ZHCT	1.719	1.367	-0.19	-0.048
75%-SR_{pr}	1.098	0.926	0.502	0.165	AN	1.665	1.276	-0.22	-0.179
90%-SR_{pr}	0.968	0.845	0.609	0.201	TW	1.795	1.415	-0.263	-0.113

sub-estimation period. Now, although `ivrv` remains the most likely factor candidates, both `omom` and `jr` also appear as distinctive and probable factors to be included in the SDF.

Although we cannot draw definite conclusions as to why the factor composition of the SDF in the options market, as reflected by the changing relative posterior probabilities described above, seems to change noticeably over time, a couple of observations are important to mention. First and foremost, many of the factors that appear to gain relevance during the second half of the sample period, namely `omom` and `jr`, tend to proxy for risks that market

Table 7: Out-of-sample pricing performance for different estimation and evaluation periods - Puts.

This table reports four performance measures of time-series out-of-sample pricing for different put factor models. In Panel A (B), we use the first (second) half of the sample period, March 1997 to June 2006 (July 2006 to December 2021), for the model estimation and recovery of risk prices and evaluate its pricing ability on the omitted half. For the BMA-SDF, prices of risk are estimated using the methodology outlined in Section 2. For the benchmark models, we use GMM with a GLS weighting matrix to estimate risk prices. Benchmark models are described in Internet Appendix IA2. *CAPM* refers to a one-factor model utilizing only `ew_ret`, whereas *51 factors* utilizes all 30 traded and 21 non-traded factors. Out-of-sample test assets are the 26 long-short factors detailed in Internet Appendix IA1.4.2 as well as 17 long portfolios based on FF17 industry sorts. Portfolio returns are calculated with equal put option weighting. RMSE and MAPE are based on returns standardized to an annual volatility of 100%.

Panel A: estimation period: March 1997 to July 2009, evaluation period: August 2009 to December 2021									
BMA-SDF	RMSE	MAPE	R^2_{ols}	R^2_{gls}	Model	RMSE	MAPE	R^2_{ols}	R^2_{gls}
10%-SR_{pr}	1.986	1.501	0.078	0.027	51 factors	5.278	4.699	-3.576	-2.625
25%-SR_{pr}	1.718	1.295	0.299	0.093	CAPM	2.129	1.625	-0.044	0.009
40%-SR_{pr}	1.467	1.127	0.499	0.163	HVX	2.372	1.771	-0.41	-0.113
60%-SR_{pr}	1.215	1	0.693	0.239	ZHCT	2.129	1.605	-0.073	0.014
75%-SR_{pr}	1.138	1.008	0.767	0.273	AN	2.118	1.618	-0.042	0.072
90%-SR_{pr}	1.25	1.129	0.748	0.281	TW	2.292	1.713	-0.362	-0.134

Panel B: estimation period: August 2009 to December 2021, evaluation period: March 1997 to July 2009									
BMA-SDF	RMSE	MAPE	R^2_{ols}	R^2_{gls}	Model	RMSE	MAPE	R^2_{ols}	R^2_{gls}
10%-SR_{pr}	1.487	1.135	0.064	0.016	51 factors	12.622	11.419	-29.222	-11.442
25%-SR_{pr}	1.397	1.085	0.14	0.056	CAPM	1.571	1.212	0.011	0.01
40%-SR_{pr}	1.427	1.15	0.159	0.091	HVX	1.566	1.258	0.098	0.012
60%-SR_{pr}	1.427	1.183	0.23	0.148	ZHCT	1.908	1.529	-0.224	-0.073
75%-SR_{pr}	1.387	1.168	0.329	0.199	AN	1.414	1.108	0.226	-0.043
90%-SR_{pr}	1.348	1.142	0.452	0.254	TW	1.885	1.51	-0.198	-0.104

makers face in the options market. The fact that these factors remain relevant and even increase in importance is in line with McLean & Pontiff (2016) who point out that the relevance of (stock) factors tends to decrease over time if factors reflect mispricing, whereas the importance of factors tends to persist if they reflect risk. Moreover, the decreasing relevance of `ivrv`, a factor that can reflect general mispricing in the options market or even overreaction by investors (Goyal & Saretto, 2009), hints at a trend of decreasingly pronounced but still existing mispricing in the options market in later periods. On the contrary, during earlier periods, general mispricing, as represented by `ivrv`, appears to have been a much more relevant driver of option returns compared to factors that reflect risk and only have posterior probabilities marginally larger than 50%.

Finally, we turn to the results for the time-series out-of-sample performance. Tables 6 and 7 reports the analogous results to Panel B in Tables 3 and 4 for calls and puts respectively. Panel A (B) shows the results for using the first (second) half of the sample period for model estimation and Panel B (A) shows the results for the second (first) subperiod as the model evaluation period. The BMA-SDF outperforms the reduced-form benchmark models on the right-hand side of the panels across all subperiod specifications for both calls and puts. Consequently, the BMA-SDF confirms its superiority vis-à-vis other benchmark models in the time-series out-of-sample test as well.

5. Additional analyses

5.1. Accounting for transaction costs

Detzel et al. (2023) find that some stock market factors with the highest returns also incur the highest trading costs. This is in line with efficiently inefficient markets because, if not for trading costs or risk-based explanations, such anomalies should be quickly arbitraged away. The authors argue that if an asset pricing model is estimated from factor returns without subtracting transaction costs, the resulting SDF cannot distinguish between true sources of risk premia, which inform the implementable optimal portfolio, and unattainable paper profits that vanish after accounting for trading frictions.

Although transaction costs in options are likely lower than what quoted spreads suggest (Muravyev & Pearson, 2020), even conservative estimates of realized transaction costs significantly reduce factor returns in our sample. This motivates estimating the BMA-SDF net of transaction costs to identify likely relevant sources of true, attainable risk premia. To mitigate the impact of transaction costs, we follow Goyal & Saretto (2024) and Zhan et al. (2022) among others, and turn away from the month-end to month-end holding periods to expiration-date to expiration-date holding periods. More precisely, we initiate option positions and construct factors each Monday after the third Friday of the month. The daily delta-hedged options are then held until maturity. Thus, no transaction costs are incurred when closing the positions. In line with the results by Muravyev & Pearson (2020), we subtract only 30% of the quoted half spread at position initiation when calculating the option profit/loss in Equation (5) for a long position. We add 30% of the quoted half spread for short positions so that the effect of increasing transaction costs in shorted options is negative when calculating long-short factor returns.

To construct factors, we assign long and short positions in the low and high decile portfolios such that the factor yields positive returns before transaction costs. Therefore, if a factor yields negative returns after transaction costs, it is not an available strategy (Goyal & Saretto, 2024). As reported in Figure C1 in the Appendix, some factors yield extremely negative Sharpe ratios after accounting for transaction costs because they are trading highly illiquid options. Since factors have to be able to price themselves when estimating the BMA-

SDF, keeping these factors in the sample will lead to some of them being assigned a high posterior inclusion probability. However, these results would be misleading for an investor who estimates the SDF to identify relevant (and profitable) sources of risk premia and not strategies that incur high transaction costs. Therefore, we exclude long-short factors from the factor set as test assets if the mean factor returns net of transaction costs are significantly negative at the 5% level.[13] For calls, we filter out nine factors of the factor set and eleven long-short out-of-sample test assets. For puts, 13 factors of the factor set and nine out-of-sample test assets are excluded.

We report posterior factor inclusion probabilities when accounting for transaction costs in Figure C2 in the Appendix. Even net of transaction costs, `ivrv` and `omom` remain the two factors with the highest inclusion probability. For puts, this probability quickly approaches one for low shrinkage, i.e., a high prior Sharpe ratio. While these factors capture general (persistent) mispricing, the results indicate that this mispricing is not solely based on trading frictions. After accounting for transaction costs, the high Sharpe ratios on `ivrv` and `omom` as well as the factors' highly probable inclusion in the SDF point towards the notion that these factors capture underlying risks for investors that keep arbitrage capital away. On the other hand, the third important factor, `jr`, yields returns close to zero for calls and is even excluded for puts after transaction costs. The factor is trading expensive options in terms of bid-ask spreads, and thus returns reflect trading frictions and limits to arbitrage, but not necessarily risk premia.[14] We report the cross-sectional pricing performance of the

[13]We follow Newey & West (1987) to calculate standard errors of the sample mean estimation robust to autocorrelation for up to four months when determining significance levels.

[14]Note that we are not optimizing factor returns after transaction costs, i.e., we do not trade-off between transaction costs and the strength of the characteristics' signals. Adopting the latter could still yield a

BMA-SDF in Tables C1 and C2. We do not exclude any factors in the low-dimensional benchmark models, even if returns are significantly negative. Again, the BMA-SDF with low shrinkage yields lower pricing errors than any of the benchmark models, both in-sample and out-of-sample.

5.2. Impact of retail investors

A considerable number of option factors are based on the hypothesis that investors in the options market exhibit behavioral biases and preferences for certain options contracts. For instance, Byun & Kim (2016) demonstrates that investor preference for lottery-like options leads to overvaluation and subsequent lower returns. Other related works such as Stein (1989), Poteshman (2001), or Goyal & Saretto (2009) discuss and investigate the effects of investor over- or underreaction in the options market. Hence, this subsection aims to assess different implications for the composition of the SDF in the options market depending on the role of behavioral influences. For this purpose, we perform our empirical analysis for options with either high or low retail investor activity. This approach is in line with the view that retail investors are more susceptible to behavioral biases in their trading and therefore impact asset returns (see e.g., Hvidkjaer, 2008; Kaniel, Saar, & Titman, 2008; Barber, Odean, & Zhu, 2008; Kumar, 2009; Han & Kumar, 2013).

We utilize signed trade and volume data from four US options exchanges operated by NASDAQ: Nasdaq GEMX (GEMX), Nasdaq International Security Exchange (ISE), Nasdaq Options Market (NOM), and Nasdaq PHLX (PHLX). These exchanges comprise a significant

profitable jump risk factor.

share of the overall US equity options market.[15] The data provides signed volumes and the number of trades (open buy/sell; close buy/sell) by non-market makers including professional customers, firm customers, and all other customers. The overall sample period covered by the data from the four exchanges ranges from May 2005 to February 2021. We only consider options with maturity between 30 and 70 days and a strike-to-spot ratio, K/S, between 0.8 and 1.2. These filters yield a sample of signed volumes that closely match the time to expiration and moneyness of the short-maturity ATM options considered in our previous analyses.

To measure the activity of retail clients in the options market, we compute the share of *"Customer - Volume of small trades"* over the total end-user volume on the four options exchanges during a month t. For both call and puts options, we only consider the volume share of retail investors for the respective option type. Although the resulting variable $RetailShare$ does not precisely reflect the share of retail investors in the options market, we consider small trades by non-professionals and non-firm customers as the volume bucket with the lowest relative investor sophistication.

We split our contract-level option data based on the median of $RetailShare$ at factor construction and rebuild option factors (and test assets) for both subsamples. Calls and puts exhibit similar average median $RetailShare$ values of 56.6% and 53.5%. For the factor return months from June 2005 to March 2021, the exchange volume data covers over 98% of the contracts in our initial options data. To obtain a suitable baseline for the high and low-

[15]In 2021, the four exchanges held a market share of roughly 25%, see data from the Options Clearing Corporation.

retail results, we determine posterior probabilities of factors using all the options contracts covered by the signed volume data for the periods starting in 2005. The resulting posterior probabilities in Figure D1 reveal one notable insight for the option factors constructed with the entire sample of options with signed volume information: `omom` is the most likely factor to be included in the SDF, whereas `ivrv` only follows with the second-highest posterior probability. To the contrary, in most of the baseline analyses, our results indicate a higher importance of `ivrv` compared to `omom`.

Next, we compute posterior factor inclusion probabilities separately for factors constructed using either high or low-*RetailShare* options. For calls and factors constructed based on options with higher retail activity in Figure D2 in the Appendix, the familiar important factors `ivrv`, `jr`, `cash_at`, and `omom` are among the most likely factors to be included in the SDF. However, compared to the results using factors constructed on all options with signed volumes in Figure D1 in the Appendix, `ivrv` is the most dominant factor in terms of posterior probability. This change of relative posterior probability hints at general mispricing, as reflected by `ivrv`, being more relevant for explaining returns of options that are more influenced by retail trading. The factor `log_price` is also among the more likely factors with a posterior probability of over 60% for larger prior Sharpe ratios. The motivation for this factor, as outlined by Boulatov et al. (2022), is that inattention leads investors regard options on low-priced stocks as unwarrantedly cheap. Assuming that less sophisticated investors being more prone to this fallacy, the `log_price` factor is arguably more important when pricing options with a high retail share.[16]

[16] On the other hand, for calls and low-*RetailShare* options, the resulting posterior probabilities are more similar to the benchmark in Figure D1 in the Appendix with `omom` representing the mostly likely candidate

Turning to puts in Figure D3 in the Appendix, we observe that `ivrv` and `jr` are the only two standout factors with large posterior probabilities. `omom` has only a comparatively low posterior probability of barely above 55% for a prior Sharpe ratio of 60% of the maximum. Again, the key role of `ivrv` in the sample with high-retail options might point to a more pronounced role of mispricing for these options. For factors based on low-$RetailShare$, `omom` emerges as the sole factor with high posterior probability by a large margin. In particular, the posterior probability of `ivrv` below 60% in any prior Sharpe ratio setting is indicative of a less central role of mispricing for puts with low retail activity. Finally, Tables D1 and D2 in the Appendix show out-of-sample tests for the BMA-SDF versus reduced-form benchmark models based on factors constructed on subsamples with high and low retail activity. In general, we verify our baseline results of superior pricing performance by the BMA-SDF for prior Sharpe Ratios of 40% of the maximum or higher, regardless of the high or low-retail split.

5.3. Robustness tests

We consider several robustness checks to show the stability of our main results. In particular, we adjust the weighting of single-option returns in the construction of option factors and test assets. Additionally, we use an alternative option return definition, i.e., margin-adjusted returns, and a more conservative prior factor inclusion probability.

Instead of assigning equal weights to single option returns, we first value-weight returns by the option contract's dollar-open interest and the market capitalization of the underlying

of being included in the SDF, followed by `ivrv` and `jr`.

stock at portfolio formation. In the spirit of Jensen et al. (2023), we limit the impact of options with extremely high dollar-open interest and market capitalization by winsorizing the weighting variables at the monthly 80^{th} percentile. Most notably, for calls and puts in Figure IA6.1, `omom` is the most likely factor included in the SDF for moderate and low degrees of shrinkage. `ivrv` appears to only play an important role for low prior Sharpe ratio values and sharply drops in terms of posterior probability after allowing for less shrinkage. For puts, both `ivrv` and `omom` exhibit by far the highest posterior probabilities.[17] We also show robustness checks for value-weights by the market capitalization of the underlying stock. For calls in Panel (a) of Figure IA6.2 in the Internet Appendix, `omom`, `ivrv`, and `jr` emerge as the most relevant factors based on their posterior probabilities. For puts in Panel (b) of Figure IA6.2 in the Internet Appendix, in addition to these three option factors, `hc` appears to be a likely candidate for being included in the SDF. The out-of-sample pricing performance vis-à-vis the benchmark factors remains robust (Tables IA6.3 and IA6.4 in the Internet Appendix).

Next, we offer robustness analyses for margin-adjusted returns as an alternative option return definition used to construct option factors and test asset returns. Margin-adjusted returns account for the margin requirements when setting up hedged long and short option positions. Details on the definition and construction of margin-adjusted option returns are displayed in Internet Appendix IA7. For calls in Panel (a) of Figure IA7.1, next to `ivrv`, `omom`, `jr`, and `cash_at` (for low degrees of shrinkage) that are important factor in our baseline

[17] As depicted in Tables IA6.1 and IA6.2 in the Internet Appendix, the BMA-SDF continues to provide superior cross-sectional out-of-sample pricing performance compared to the benchmark reduced-form factor models.

estimation in Section 4.1, the equal-weighted option market factor, `ew_ret`, displays a posterior probability considerably above the prior of 50%. Also, the embedded leverage factor, `embedlev`, displays large posterior probabilities for high and moderate shrinkage levels. The reason for the significance of this factor in the SDF might be due to the mechanical relation between margin-adjusted returns and embedded leverage (options with high embedded leverage typically have higher margin requirements because they present a greater risk to the investor) and margins are the highest for short call positions due to their unlimited loss potential. For puts in Panel (b) of Figure IA7.1, `ivrv`, `omom`, and `jr` are the most probable factors with high posterior probabilities. As can be seen in Tables IA7.1 and IA7.2, the BMA-SDF outperforms the benchmark factor models similarly to our baseline results.

Finally, we follow Dickerson et al. (2023) and conduct the main analysis with more conservative prior beliefs about model dimensionality, drawing initial factor inclusion probabilities from a $Beta(3, 12)$ distribution, which translates to an expected inclusion of 20% of factors. Results are reported in Internet Appendix IA8. As shown in Figure IA8.1, the same four calls factors and put factors that emerged in the main analysis emerge as likely SDF candidates. The posterior probabilities for the other factors are around or below the initial 20%, which reflects the prior assumption of sparsity. However, while still beating the low-dimensional benchmark models with low regularization, results in Tables IA8.1 and IA8.2 suggest that out-of-sample pricing performance is worse than when estimating the BMA-SDF by drawing factor inclusion probabilities from a $Beta(1, 1)$ distribution, providing further evidence that the true SDF for equity options is dense in nature.

In summary, despite some minor changes in the posterior factor probabilities, our robustness checks show that the BMA-SDF's cross-sectional pricing performance remains robust across different return weighting specifications and option return definitions.

6. Conclusion

Using the Bayesian method proposed by Bryzgalova, Huang, & Julliard (2023), we estimate posterior risk prices and probabilities of factors included in the SDF that prices the cross-section of delta-hedged option returns. Considering a set of 30 traded option factors and 15 nontraded factors in combination with six widely established stock factors, we find that the difference between implied and realized volatility, option return momentum, and jump risk are included in the SDF with high probability. To a lesser extent, other factors such as the cash-to-assets ratio or idiosyncratic volatility are further factor candidates that span the risks driving option returns. Similar to the results for the stock market, we observe that the SDF is dense in the space of observable option factors, with the average number of factors included in the SDF being close to 25 for high and moderate levels of shrinkage.

We show that the estimated Bayesian model averaging SDF (BMA-SDF) exhibits superior cross-sectional out-of-sample pricing performance compared to reduced-form option benchmark models such as the model proposed by Horenstein et al. (2022) or Tian & Wu (2023), pricing 26 additional option anomalies and 17 industry-sorted option portfolios. A reduced-form four-factor model based on the exterior probability implied by the BMA method also outperforms existing benchmark models out-of-sample. Finally, implied-minus-realized volatility and option momentum remain important factors in the SDF after transaction costs and mispricing tends to be more pronounced in subsamples that construct factors and test assets based on options with high retail activity.

Primarily, our paper contributes to the nascent literature on factors explaining the cross-section of option returns. Our empirical results verify the relevance of factors such as the difference between implied and realized volatility, jump risk, and option momentum in previously proposed models such as Horenstein et al. (2022) or Tian & Wu (2023). At the same time, the dense model dimensionality implied by the option BMA-SDF highlights the benefits of including more than three to four factors in linear option factor models to account for multiple sources of imperfectly identified sources of risk.

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A Posterior factor inclusion probabilities and risk prices

Table A1: Posterior factor inclusion probabilities and prices of risk - Calls. This table lists posterior factor inclusion probabilities $\mathbb{E}[\gamma_j|\text{data}]$ and posterior prices of risk $\mathbb{E}[\lambda_j|\text{data}]$ for each factor j. The continuous spike-and-slab BMA approach is detailed in Section 2. We employ non-informative prior beliefs about factor inclusion, drawing factor inclusion probabilities from a $Beta(1,1)$ distribution. Prior annualized Sharpe ratios range from 10% to 90% of the ex-post maximum achievable Sharpe ratio. The factor set includes returns of 30 traded long-short factors based on delta-hedged call returns as well as 21 non-traded factors from February 1997 to December 2021. Detailed descriptions of the characteristics used for factor construction are documented Internet in Appendix IA1.1. Additional test assets are 5×5 long portfolios based on independent monthly sorts on `ivrv` and `doi`. Portfolio returns are calculated with equal call option weighting.

	Post. Factor Inclusion Probability						Post. Price of Risk					
	Prior Sharpe Ratio (% of max)						Prior Sharpe Ratio (% of max)					
Factors:	10%	25%	40%	60%	75%	90%	10%	25%	40%	60%	75%	90%
ivrv	0.525	0.624	0.737	0.822	0.858	0.856	-0.011	-0.074	-0.187	-0.372	-0.527	-0.702
omom	0.507	0.575	0.669	0.763	0.805	0.824	0.007	0.045	0.120	0.262	0.393	0.557
jr	0.506	0.556	0.612	0.674	0.688	0.653	-0.005	-0.035	-0.089	-0.181	-0.252	-0.308
cash_at	0.506	0.507	0.523	0.558	0.591	0.613	-0.007	-0.034	-0.073	-0.155	-0.255	-0.419
CMA	0.491	0.498	0.506	0.511	0.524	0.536	-0.001	-0.004	-0.010	-0.026	-0.049	-0.104
TERM	0.495	0.497	0.502	0.513	0.517	0.524	-0.001	-0.004	-0.011	-0.024	-0.043	-0.089
log_price	0.501	0.515	0.512	0.496	0.504	0.505	0.008	0.037	0.064	0.104	0.161	0.282
defrisk	0.495	0.483	0.479	0.481	0.495	0.504	-0.001	0.005	0.025	0.072	0.133	0.252
SKEW	0.497	0.498	0.498	0.500	0.497	0.499	0.000	0.001	0.001	0.004	0.007	0.017
CPTL	0.496	0.492	0.497	0.496	0.504	0.492	0.001	0.006	0.015	0.035	0.063	0.113
Mom	0.501	0.501	0.498	0.498	0.499	0.487	-0.000	-0.000	-0.000	-0.000	-0.001	-0.001
LIQ	0.497	0.497	0.498	0.494	0.491	0.482	0.000	0.000	0.001	0.003	0.006	0.008
SKEWTS	0.495	0.495	0.494	0.493	0.489	0.478	-0.000	-0.000	-0.001	-0.002	-0.004	-0.012
ICRC	0.498	0.496	0.497	0.496	0.497	0.478	0.001	0.004	0.011	0.027	0.048	0.082
HML	0.494	0.493	0.492	0.489	0.489	0.474	-0.000	-0.002	-0.005	-0.012	-0.021	-0.036
EPU	0.492	0.491	0.495	0.498	0.488	0.470	-0.000	-0.002	-0.006	-0.014	-0.023	-0.041
RMW	0.491	0.492	0.495	0.490	0.482	0.468	-0.000	-0.001	-0.002	-0.005	-0.007	-0.009
DEF	0.496	0.494	0.494	0.483	0.478	0.463	-0.000	-0.001	-0.001	-0.004	-0.010	-0.024
Mkt-RF	0.495	0.492	0.497	0.495	0.488	0.463	0.001	0.007	0.017	0.040	0.064	0.092
SMB	0.495	0.494	0.488	0.486	0.478	0.458	0.000	0.001	0.001	-0.002	-0.008	-0.026
SENT	0.496	0.493	0.492	0.485	0.474	0.451	0.001	0.005	0.009	0.010	0.004	-0.017
UNCr	0.495	0.493	0.489	0.482	0.477	0.449	0.001	0.004	0.009	0.015	0.019	0.027
VIX	0.503	0.499	0.499	0.494	0.480	0.444	-0.002	-0.010	-0.024	-0.051	-0.076	-0.103
STLFSI	0.491	0.497	0.493	0.479	0.465	0.440	0.001	0.006	0.013	0.016	0.009	-0.012
ivol	0.491	0.482	0.473	0.468	0.457	0.437	-0.007	-0.021	-0.032	-0.063	-0.108	-0.188
UNC	0.495	0.494	0.493	0.480	0.466	0.436	0.001	0.005	0.010	0.013	0.010	-0.004
hvol	0.494	0.483	0.473	0.456	0.445	0.422	-0.005	-0.009	-0.007	-0.021	-0.057	-0.127
UNCf	0.492	0.491	0.480	0.470	0.453	0.415	-0.001	-0.003	-0.005	-0.010	-0.015	-0.025
VIXVOL	0.497	0.493	0.490	0.474	0.455	0.412	0.000	0.002	0.004	0.001	-0.008	-0.020
optspread	0.494	0.487	0.479	0.460	0.444	0.408	0.000	0.004	0.014	0.037	0.062	0.097
tskew	0.496	0.485	0.474	0.454	0.439	0.405	-0.001	0.002	0.013	0.039	0.068	0.108
amihud	0.495	0.489	0.464	0.445	0.428	0.404	-0.004	-0.011	-0.007	0.019	0.055	0.113
zscore	0.499	0.494	0.486	0.466	0.444	0.399	0.002	0.010	0.021	0.037	0.053	0.085
netis_at	0.502	0.485	0.465	0.445	0.425	0.395	-0.005	-0.011	-0.008	0.010	0.042	0.108
sysvol	0.490	0.483	0.469	0.439	0.416	0.390	0.001	0.014	0.028	0.016	-0.029	-0.117
max10	0.500	0.483	0.472	0.453	0.435	0.386	-0.003	-0.001	0.014	0.026	0.024	0.022
embedlev	0.507	0.513	0.504	0.472	0.428	0.380	0.012	0.044	0.068	0.074	0.053	0.002
ivterm	0.499	0.502	0.494	0.470	0.438	0.375	-0.005	-0.023	-0.040	-0.054	-0.061	-0.070
iskew	0.497	0.491	0.477	0.453	0.426	0.372	-0.000	0.002	0.009	0.023	0.036	0.046
hc	0.504	0.502	0.477	0.442	0.412	0.368	-0.007	-0.025	-0.032	-0.023	-0.005	0.016
rsi	0.495	0.480	0.471	0.447	0.422	0.366	-0.001	-0.000	0.009	0.032	0.056	0.077
ocfq_saleq_std	0.507	0.522	0.522	0.504	0.454	0.365	-0.010	-0.044	-0.077	-0.110	-0.114	-0.090
ac	0.493	0.493	0.481	0.466	0.434	0.361	0.001	0.006	0.018	0.038	0.052	0.056
vov	0.496	0.496	0.485	0.459	0.426	0.361	-0.001	-0.007	-0.011	-0.012	-0.006	0.011
vr	0.505	0.514	0.504	0.474	0.430	0.353	-0.010	-0.038	-0.061	-0.076	-0.071	-0.049
ope_be	0.499	0.518	0.508	0.483	0.437	0.351	0.010	0.039	0.066	0.087	0.083	0.047
ebit_sale	0.506	0.523	0.513	0.473	0.422	0.338	0.010	0.042	0.069	0.077	0.055	0.005
issue_1y	0.497	0.479	0.461	0.433	0.400	0.326	-0.004	-0.009	-0.009	-0.008	-0.004	0.006
disp	0.496	0.481	0.467	0.430	0.398	0.325	-0.003	-0.006	-0.002	0.010	0.022	0.035
ew_ret	0.490	0.466	0.446	0.402	0.371	0.316	-0.000	0.011	0.028	0.041	0.048	0.059
issue_5y	0.496	0.492	0.469	0.435	0.392	0.314	-0.006	-0.019	-0.026	-0.025	-0.017	0.001

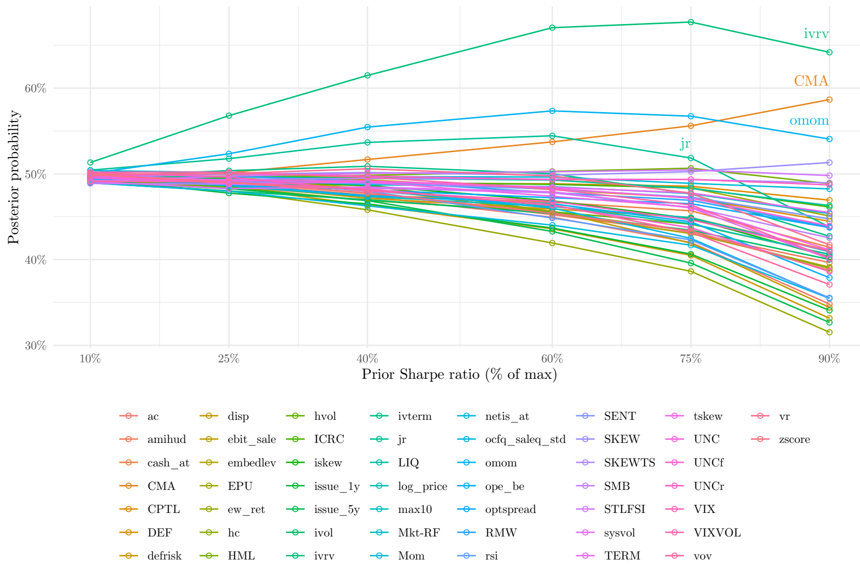
Table A2: Posterior factor inclusion probabilities and prices of risk - Puts.

This table lists posterior factor inclusion probabilities $\mathbb{E}[\gamma_j|\text{data}]$ and posterior prices of risk $\mathbb{E}[\lambda_j|\text{data}]$ for each factor j. The continuous spike-and-slab BMA approach is detailed in Section 2. We employ non-informative prior beliefs about factor inclusion, drawing factor inclusion probabilities from a $Beta(1,1)$ distribution. Prior annualized Sharpe ratios range from 10% to 90% of the ex-post maximum achievable Sharpe ratio. The factor set includes returns of 30 traded long-short factors based on delta-hedged put returns as well as 21 non-traded factors from February 1997 to December 2021. Detailed descriptions of the characteristics used for factor construction are documented in Internet Appendix IA1.1. Additional test assets are 5×5 long portfolios based on independent monthly sorts on `ivrv` and `doi`. Portfolio returns are calculated with equal put option weighting.

	Post. Factor Inclusion Probability						Post. Price of Risk					
	Prior Sharpe Ratio (% of max)						Prior Sharpe Ratio (% of max)					
Factors:	10%	25%	40%	60%	75%	90%	10%	25%	40%	60%	75%	90%
ivrv	0.521	0.643	0.783	0.887	0.926	0.951	-0.012	-0.086	-0.231	-0.479	-0.696	-0.993
jr	0.507	0.566	0.643	0.729	0.755	0.725	-0.007	-0.046	-0.120	-0.249	-0.346	-0.413
omom	0.503	0.557	0.637	0.706	0.724	0.714	0.006	0.041	0.107	0.218	0.306	0.396
ivol	0.503	0.510	0.518	0.544	0.586	0.645	-0.011	-0.044	-0.082	-0.175	-0.311	-0.601
SKEW	0.497	0.499	0.497	0.498	0.500	0.503	0.000	0.001	0.003	0.006	0.011	0.026
SKEWTS	0.495	0.497	0.495	0.492	0.496	0.488	0.000	0.000	0.001	0.003	0.007	0.022
Mom	0.501	0.497	0.500	0.496	0.492	0.484	-0.000	-0.001	-0.003	-0.006	-0.011	-0.027
SENT	0.495	0.492	0.485	0.485	0.482	0.482	0.000	0.003	0.004	-0.000	-0.014	-0.050
cash_at	0.505	0.504	0.507	0.511	0.511	0.482	-0.008	-0.033	-0.065	-0.121	-0.177	-0.246
TERM	0.493	0.496	0.496	0.491	0.485	0.477	-0.000	-0.002	-0.005	-0.009	-0.012	-0.018
CMA	0.491	0.490	0.491	0.494	0.486	0.477	-0.000	-0.001	-0.001	-0.003	-0.005	-0.001
HML	0.494	0.489	0.489	0.491	0.483	0.475	-0.000	-0.001	-0.003	-0.007	-0.014	-0.027
DEF	0.496	0.492	0.490	0.495	0.484	0.474	0.000	0.001	0.004	0.011	0.021	0.047
LIQ	0.495	0.497	0.501	0.494	0.485	0.469	-0.000	-0.001	-0.001	-0.002	-0.003	-0.007
RMW	0.491	0.489	0.496	0.491	0.486	0.465	-0.000	-0.001	-0.003	-0.007	-0.011	-0.008
UNC	0.495	0.495	0.492	0.484	0.482	0.459	0.000	0.000	-0.003	-0.014	-0.035	-0.080
Mkt-RF	0.495	0.492	0.493	0.487	0.478	0.458	0.001	0.006	0.016	0.034	0.055	0.093
ICRC	0.498	0.492	0.489	0.483	0.473	0.457	0.000	-0.000	-0.001	-0.006	-0.016	-0.055
UNCr	0.494	0.494	0.492	0.480	0.473	0.452	0.000	0.002	0.003	-0.000	-0.008	-0.028
VIX	0.502	0.503	0.503	0.492	0.484	0.451	-0.002	-0.013	-0.031	-0.061	-0.091	-0.142
CPTL	0.496	0.495	0.489	0.483	0.476	0.450	0.001	0.004	0.008	0.014	0.015	0.004
UNCf	0.494	0.497	0.500	0.502	0.488	0.446	-0.002	-0.013	-0.029	-0.059	-0.087	-0.119
EPU	0.493	0.494	0.492	0.484	0.473	0.444	-0.000	-0.002	-0.006	-0.012	-0.019	-0.029
SMB	0.494	0.493	0.491	0.485	0.475	0.442	0.000	0.003	0.007	0.014	0.020	0.021
embedlev	0.506	0.513	0.491	0.435	0.411	0.439	0.012	0.044	0.057	0.026	-0.040	-0.214
STLFSI	0.491	0.493	0.488	0.485	0.463	0.431	0.001	0.007	0.016	0.028	0.034	0.038
defrisk	0.496	0.488	0.468	0.455	0.446	0.412	-0.002	-0.002	0.010	0.043	0.082	0.135
hvol	0.502	0.491	0.479	0.460	0.444	0.409	-0.008	-0.021	-0.025	-0.037	-0.049	-0.036
VIXVOL	0.498	0.496	0.486	0.472	0.453	0.405	0.001	0.008	0.019	0.034	0.043	0.048
optspread	0.493	0.491	0.484	0.468	0.455	0.400	0.001	0.007	0.022	0.050	0.074	0.095
max10	0.498	0.480	0.465	0.447	0.425	0.383	-0.006	-0.011	-0.004	-0.007	-0.020	-0.036
zscore	0.499	0.490	0.489	0.466	0.441	0.383	0.002	0.011	0.025	0.043	0.060	0.082
log_price	0.497	0.502	0.490	0.459	0.429	0.366	0.007	0.030	0.046	0.060	0.072	0.088
ebit_sale	0.510	0.536	0.543	0.516	0.457	0.363	0.012	0.052	0.092	0.123	0.121	0.101
vov	0.497	0.498	0.492	0.467	0.433	0.361	-0.002	-0.009	-0.017	-0.022	-0.018	-0.006
hc	0.504	0.511	0.496	0.458	0.423	0.359	-0.008	-0.033	-0.049	-0.053	-0.053	-0.062
iskew	0.498	0.487	0.473	0.451	0.417	0.358	-0.001	-0.001	0.005	0.020	0.035	0.046
vr	0.506	0.535	0.529	0.495	0.447	0.358	-0.012	-0.050	-0.083	-0.105	-0.099	-0.067
ocfq_saleq_std	0.508	0.527	0.530	0.499	0.453	0.356	-0.011	-0.048	-0.083	-0.113	-0.115	-0.095
amihud	0.498	0.491	0.472	0.438	0.411	0.355	-0.005	-0.018	-0.021	-0.008	0.009	0.027
ope_be	0.500	0.518	0.507	0.474	0.431	0.349	0.011	0.044	0.070	0.089	0.090	0.076
ivterm	0.499	0.504	0.496	0.456	0.413	0.342	-0.005	-0.022	-0.037	-0.038	-0.022	0.011
ac	0.494	0.486	0.474	0.449	0.410	0.340	-0.001	-0.001	0.001	0.007	0.012	0.014
issue_5y	0.493	0.492	0.480	0.450	0.417	0.340	-0.006	-0.021	-0.035	-0.053	-0.066	-0.074
netis_at	0.503	0.488	0.469	0.439	0.402	0.340	-0.006	-0.018	-0.019	-0.013	0.002	0.017
sysvol	0.492	0.482	0.479	0.456	0.415	0.340	0.000	0.017	0.045	0.062	0.047	0.002
tskew	0.500	0.491	0.473	0.439	0.404	0.334	-0.002	-0.004	-0.001	0.009	0.018	0.021
rsi	0.499	0.504	0.498	0.469	0.430	0.332	-0.004	-0.020	-0.036	-0.053	-0.057	-0.043
issue_1y	0.497	0.480	0.464	0.430	0.393	0.321	-0.005	-0.011	-0.012	-0.015	-0.018	-0.020
disp	0.498	0.487	0.463	0.424	0.383	0.307	-0.004	-0.010	-0.008	0.002	0.012	0.020
ew_ret	0.492	0.456	0.430	0.385	0.350	0.295	-0.003	0.001	0.012	0.022	0.029	0.032

B Different estimation periods

(a) Estimation period: March 1997 to July 2009 (1^{st} half of sample period)



(b) Estimation period: August 2009 to December 2021 (2^{nd} half of sample period)

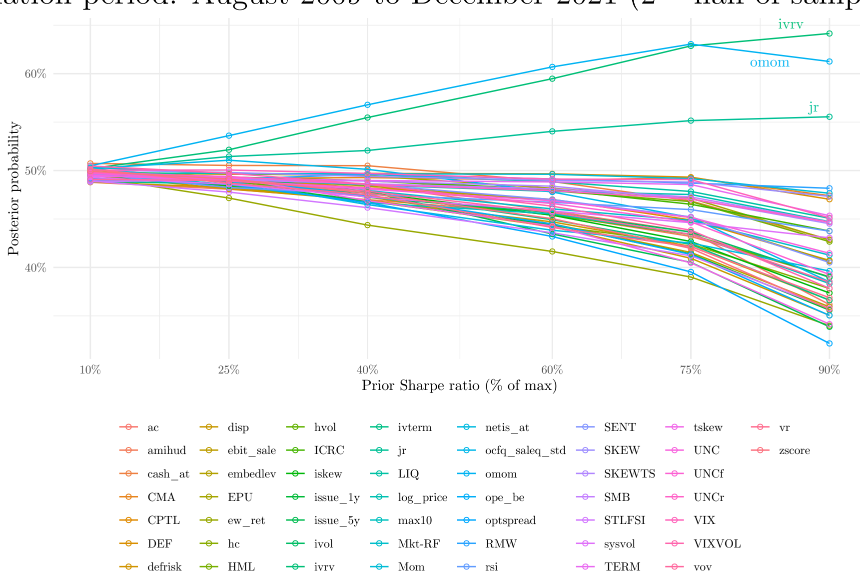
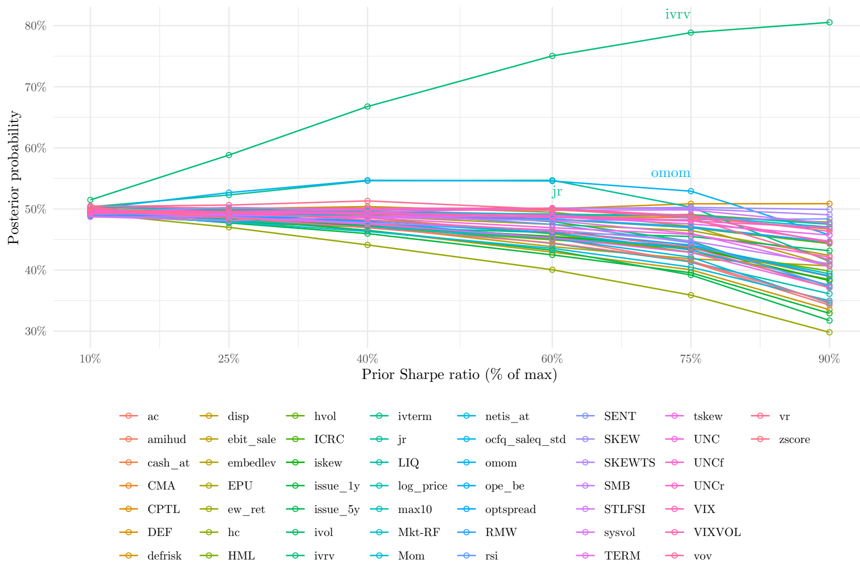


Fig. B1. Posterior factor inclusion probabilities for different estimation periods - Calls.

Notes: This figure shows posterior factor probabilities $\mathbb{E}[\gamma_j|\text{data}]$ estimated with the BMA approach outlined in Section 2 for different estimation periods. Traded test assets are based on calls. Returns for estimation over the first half of the sample period range from March 1997 to June 2006. Returns for estimation over the second half of the sample period range from July 2006 to December 2021. All other specifications are detailed in Figure 1.

(a) Estimation period: March 1997 to July 2009 (1^{st} half of sample period)



(b) Estimation period: August 2009 to December 2021 (2^{nd} half of sample period)

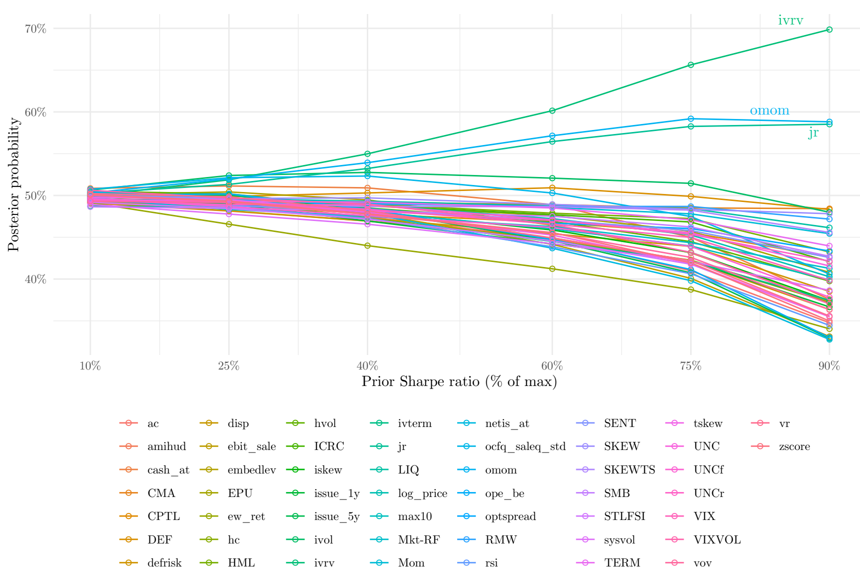
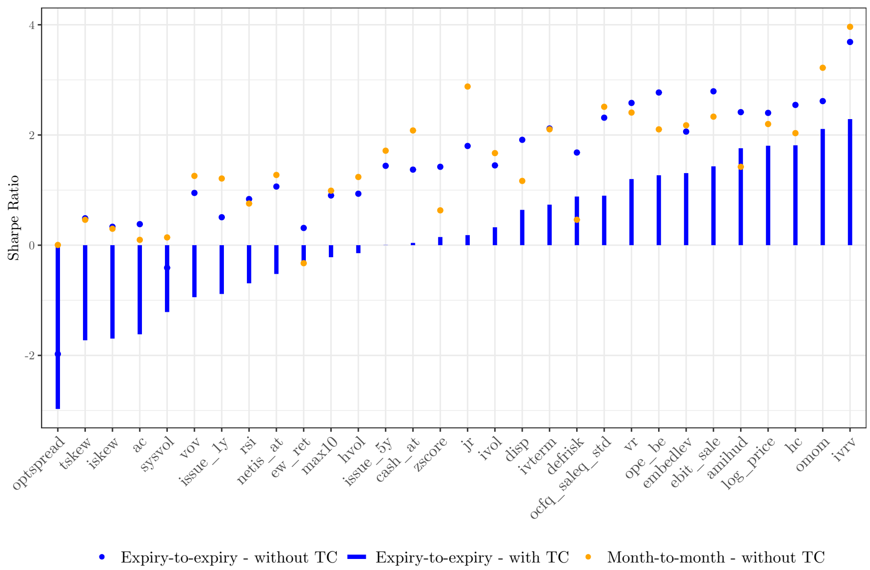


Fig. B2. Posterior factor inclusion probabilities for different estimation periods - Puts.

Notes: This figure shows posterior factor probabilities $\mathbb{E}[\gamma_j|\text{data}]$ estimated with the BMA approach outlined in Section 2 for different estimation periods. Traded test assets are based on puts. Returns for estimation over the first half of the sample period range from March 1997 to June 2006. Returns for estimation over the second half of the sample period range from July 2006 to December 2021. All other specifications are detailed in Figure 2.

C Results with transaction costs

(a) Calls



(b) Puts

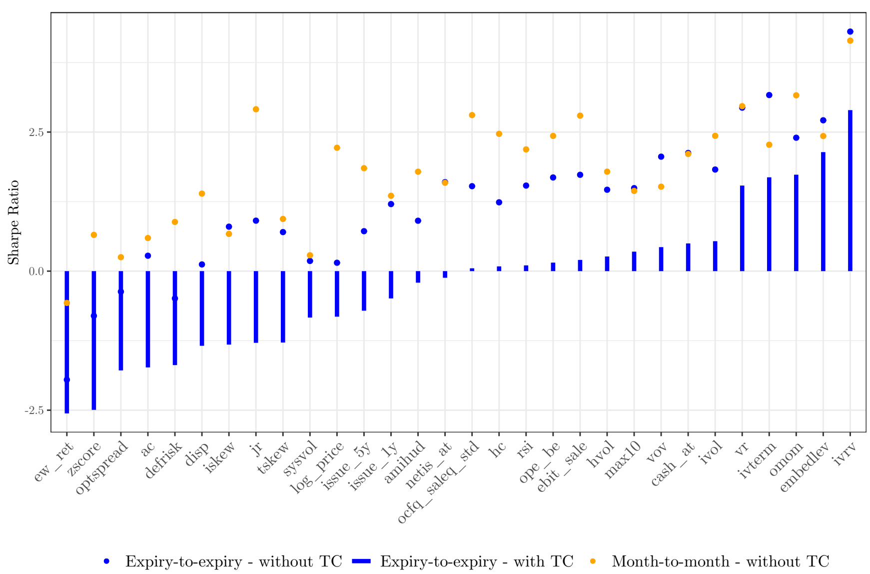


Fig. C1. Ann. Sharpe ratios of factors before/after transaction costs.
Notes: This figure shows annualized Sharpe ratios of the factor set. Factors based on call options are shown in (a) and factors based on put options in (b). The orange dots show the annualized Sharpe ratios of the standard factors constructed from daily delta-hedged options held over the last full calendar month before option maturity and transaction costs not considered. The blue dots show Sharpe ratios of factors where the underlying options are held until maturity. The new options positions (and the portfolio sorts) are initiated on the first Monday after the expiration of the old positions. The blue lines show Sharpe ratios for factors with the same expiry-to-expiry holding periods, but subtracting 30% of the spread between option price and option mid-price at position initiation as outlined in Section 5.1.

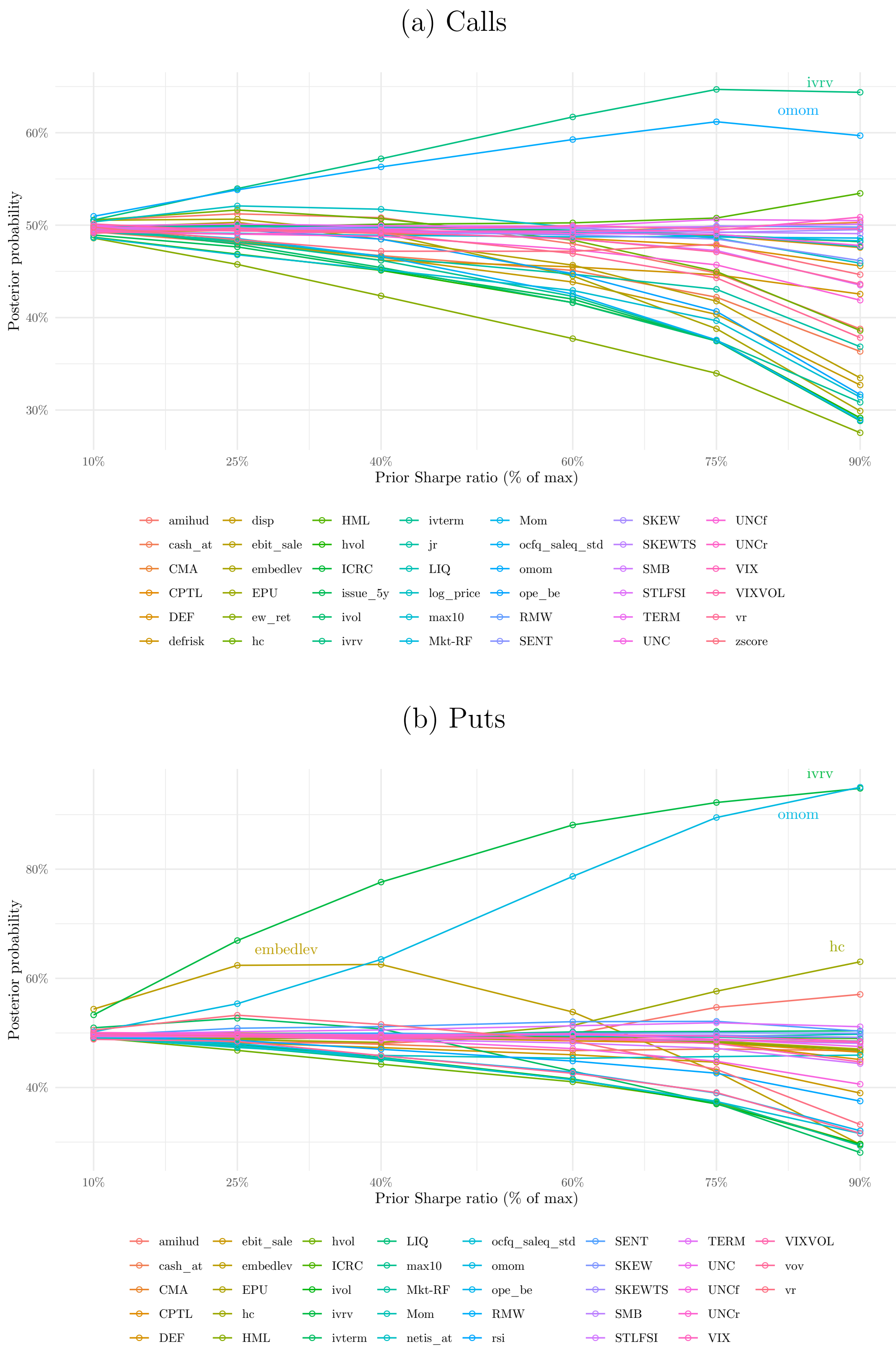


Fig. C2. Posterior factor inclusion probabilities when accounting for transaction costs.

Notes: This figure shows posterior factor probabilities $\mathbb{E}[\gamma_j|\text{data}]$ estimated with the BMA approach outlined in Section 2 for different estimation periods. To account for transaction cost, we calculate monthly option returns from expiration to expiration and subtract 30% of the spread between the option price and the option mid-price at position initiation as outlined in Section 5.1. Factor and test asset returns are then calculated by equally weighting the option returns after transaction costs. Results for calls are shown in (a) and for puts in (b). All other specifications are detailed in Figure 1.

Table C1: Cross-sectional pricing performance - Calls, accounting for transaction costs.

This table reports four performance measures of cross-sectional pricing for different factor models. For the BMA-SDF, prices of risk are estimated using the methodology outlined in Section 2. For the benchmark models, we use GMM with a GLS weighting matrix to estimate risk prices. Benchmark models are described in Internet Appendix IA2. *CAPM* refers to a one-factor model utilizing only `ew_ret`, whereas *42 factors* utilizes 21 traded and 21 non-traded factors. For the latter benchmark model, for the BMA-SDF estimation, and for the out-of-sample test assets, we exclude factors with negative mean returns on the 5% significance level using Newey & West (1987) standard errors accounting for autocorrelation up to lag 4. Out-of-sample test assets are then 15 of the long-short factors detailed in Internet Appendix IA1.4.2 as well as 17 long portfolios based on FF17 industry sorts. To account for transaction cost, we calculate monthly call option returns from expiration to expiration and subtract 30% of the quoted half-spread at position initiation as outlined in Section 5.1. Factor and test asset returns are then calculated by equally weighting the option returns after transaction costs. RMSE and MAPE are based on returns standardized to an annual volatility of 100%.

Panel A: In-Sample Pricing, Test Assets: 42 Factors and 25 IVRV-DOI portfolios									
BMA-SDF	RMSE	MAPE	R^2_{ols}	R^2_{gls}	Model	RMSE	MAPE	R^2_{ols}	R^2_{gls}
10%-SR_{pr}	1.129	0.849	0.106	0.151	51 factors	0.096	0.06	0.993	0.968
25%-SR_{pr}	0.922	0.691	0.383	0.204	CAPM	1.203	0.923	0.041	0.136
40%-SR_{pr}	0.769	0.574	0.557	0.252	HVX	0.726	0.55	0.624	0.254
60%-SR_{pr}	0.645	0.477	0.68	0.314	ZHCT	0.706	0.487	0.62	0.211
75%-SR_{pr}	0.579	0.417	0.741	0.365	AN	0.901	0.75	0.514	0.277
90%-SR_{pr}	0.525	0.368	0.788	0.427	TW	0.622	0.465	0.754	0.341

Panel B: Out-of-Sample Pricing, Test Assets: 15 Factors and 17 Industry Portfolios									
BMA-SDF	RMSE	MAPE	R^2_{ols}	R^2_{gls}	Model	RMSE	MAPE	R^2_{ols}	R^2_{gls}
10%-SR_{pr}	0.808	0.638	0.138	-0.088	42 factors	1.697	1.351	-2.801	-5.392
25%-SR_{pr}	0.645	0.479	0.45	0.015	CAPM	0.823	0.662	0.105	-0.136
40%-SR_{pr}	0.553	0.406	0.596	0.092	HVX	0.576	0.439	0.562	0.093
60%-SR_{pr}	0.506	0.37	0.663	0.157	ZHCT	0.607	0.434	0.513	0.108
75%-SR_{pr}	0.49	0.365	0.683	0.19	AN	0.748	0.554	0.26	-0.18
90%-SR_{pr}	0.483	0.367	0.692	0.204	TW	0.634	0.484	0.469	0.147

Table C2: Cross-sectional pricing performance - Puts, accounting for transaction costs.

This table reports four performance measures of cross-sectional pricing for different factor models. For the BMA-SDF, prices of risk are estimated using the methodology outlined in Section 2. For the benchmark models, we use GMM with a GLS weighting matrix to estimate risk prices. Benchmark models are described in Appendix IA2. *CAPM* refers to a one-factor model utilizing only `ew_ret`, whereas *38 factors* utilizes 17 traded and 21 non-traded factors. For the latter benchmark model, for the BMA-SDF estimation, and for the out-of-sample test assets, we exclude factors with negative mean returns on the 5% significance level using Newey & West (1987) standard errors accounting for autocorrelation up to lag 4. Out-of-sample test assets are then 17 of the long-short factors detailed in Appendix IA1.4.2 as well as 17 long portfolios based on FF17 industry sorts. To account for transaction cost, we calculate monthly put option returns from expiration to expiration and subtract 30% of the quoted half-spread at position initiation as outlined in Section 5.1. Factor and test asset returns are then calculated by equally weighting the call option returns after transaction costs. RMSE and MAPE are based on returns standardized to an annual volatility of 100%.

Panel A: In-Sample Pricing, Test Assets: 38 Factors and 25 IVRV-DOI portfolios									
BMA-SDF	RMSE	MAPE	R^2_{ols}	R^2_{gls}	Model	RMSE	MAPE	R^2_{ols}	R^2_{gls}
10%-SR_{pr}	1.518	1.235	0.139	0.218	51 factors	0.109	0.067	0.996	0.97
25%-SR_{pr}	1.145	0.853	0.505	0.32	CAPM	1.64	1.333	-0.009	0.192
40%-SR_{pr}	0.849	0.584	0.722	0.418	HVX	0.775	0.623	0.825	0.434
60%-SR_{pr}	0.603	0.406	0.857	0.541	ZHCT	1.39	0.997	0.284	0.214
75%-SR_{pr}	0.475	0.338	0.911	0.636	AN	1.301	1.037	0.335	0.243
90%-SR_{pr}	0.369	0.277	0.947	0.724	TW	0.708	0.586	0.883	0.678

Panel B: Out-of-Sample Pricing, Test Assets: 17 Factors and 17 Industry Portfolios									
BMA-SDF	RMSE	MAPE	R^2_{ols}	R^2_{gls}	Model	RMSE	MAPE	R^2_{ols}	R^2_{gls}
10%-SR_{pr}	1.293	1.036	0.151	0.356	38 factors	0.617	0.485	0.807	0.471
25%-SR_{pr}	0.971	0.731	0.522	0.427	CAPM	1.397	1.142	0.01	0.343
40%-SR_{pr}	0.762	0.554	0.706	0.476	HVX	0.79	0.601	0.684	0.319
60%-SR_{pr}	0.632	0.466	0.798	0.526	ZHCT	1.087	0.845	0.4	0.403
75%-SR_{pr}	0.573	0.428	0.833	0.568	AN	1.165	0.87	0.311	0.251
90%-SR_{pr}	0.526	0.391	0.859	0.615	TW	0.688	0.566	0.76	0.562

D Impact of retail investors

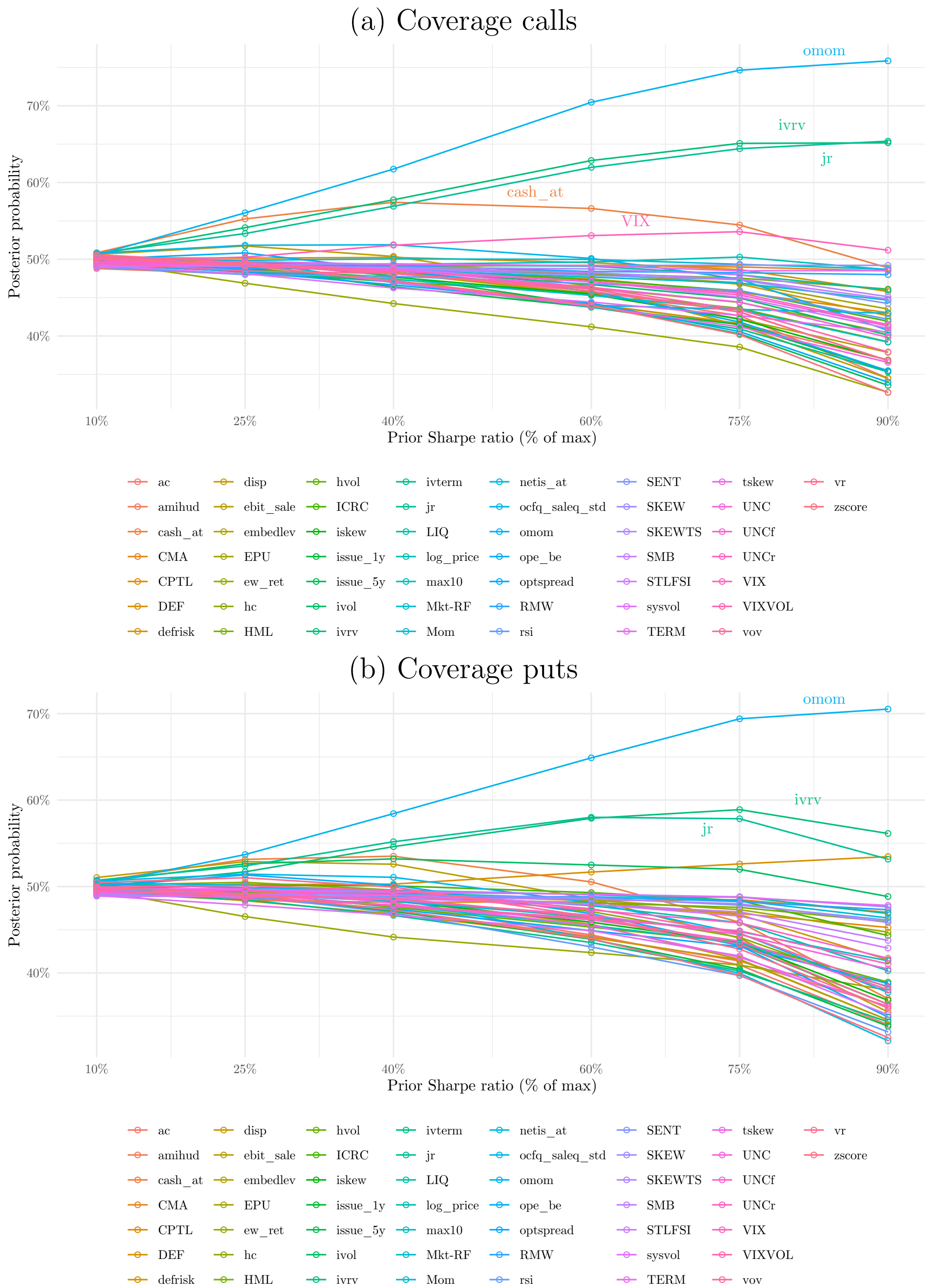
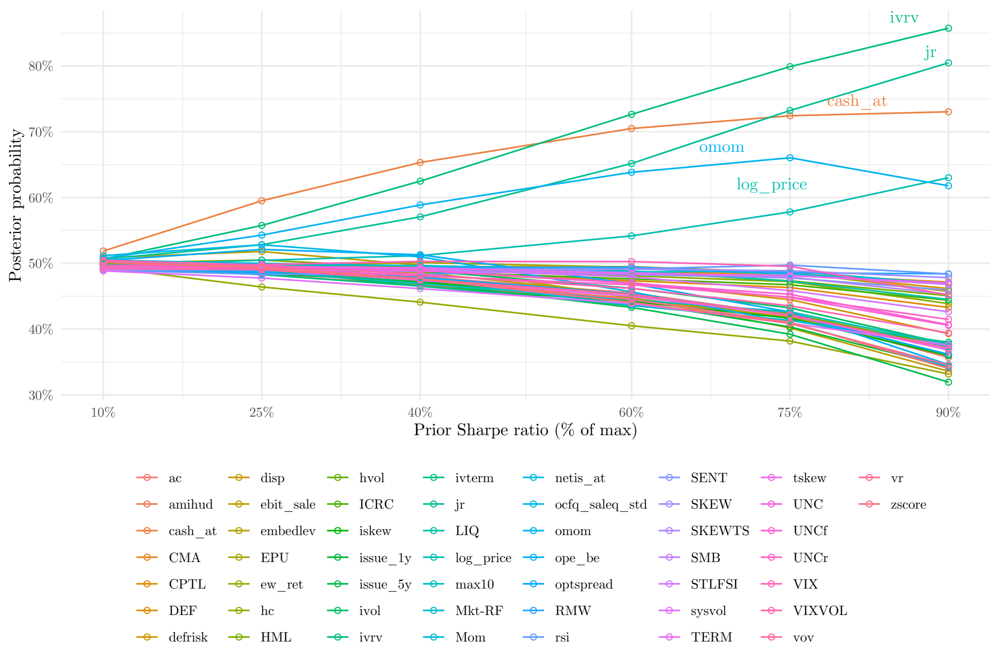


Fig. D1. Coverages (retail).

Notes: This figure shows posterior factor probabilities $\mathbb{E}[\gamma_j|\text{data}]$ estimated with the BMA approach outlined in Section 2. We only construct factors for which we have signed exchange volume data as outlined in Section 5.2. Traded test assets are based on calls and also constructed using options with signed volume data coverage. The sample period is from June 2005 to March 2021. All other specifications are detailed in Figure 1.

(a) High retail



(b) Low retail

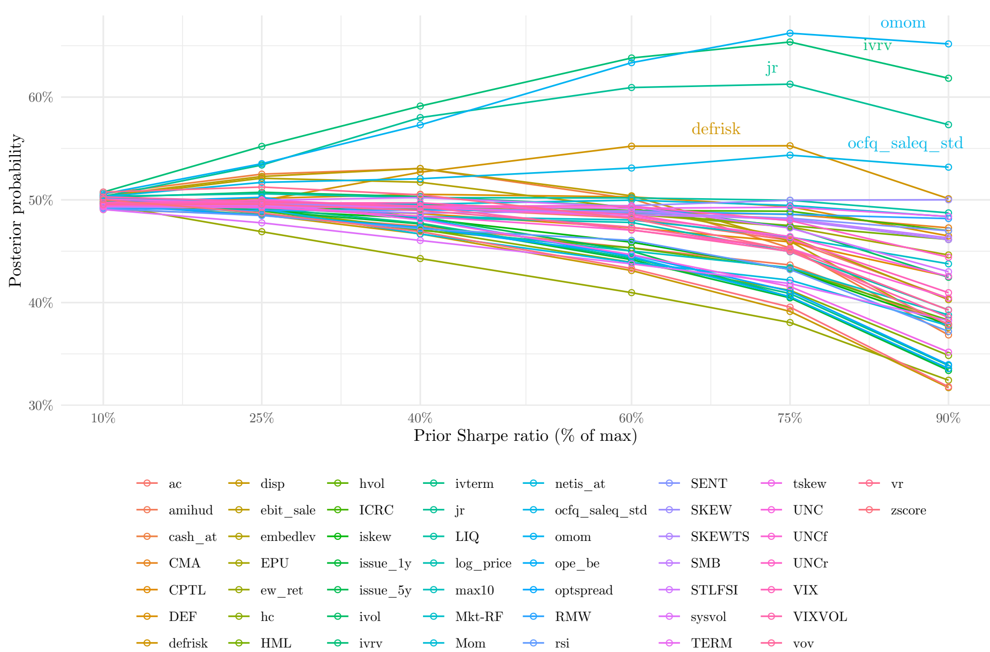
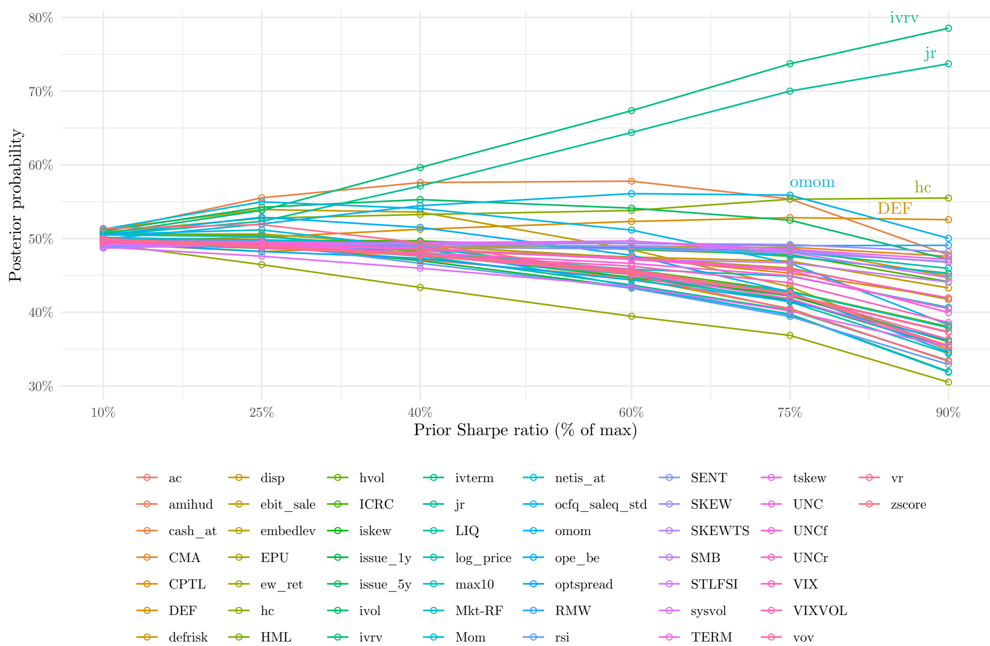


Fig. D2. Retail - Calls.

Notes: This figure shows posterior factor probabilities $\mathbb{E}[\gamma_j|\text{data}]$ estimated with the BMA approach outlined in Section 2 for factors constructed based on call options with high or low retail activity. At the beginning of each month, we sort calls based on the median of small customer volume over total non-market-maker volume and construct factors and traded option test assets for both subsamples. Traded test assets are based on calls. The sample period is from June 2005 to March 2021. All other specifications are detailed in Figure 1.

(a) High retail



(b) Low retail

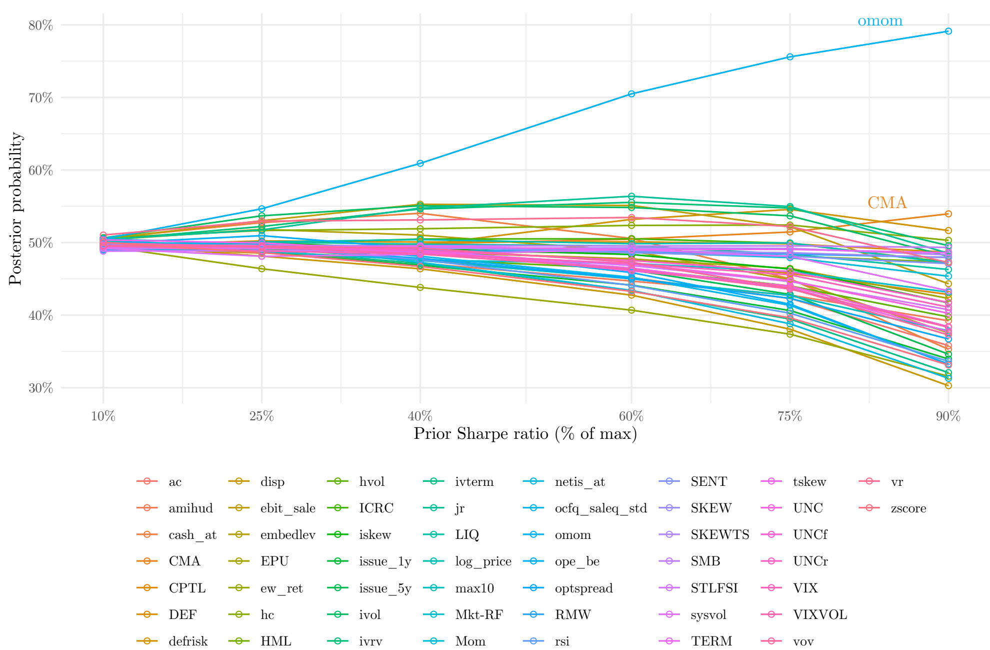


Fig. D3. Retail - Puts.

This figure shows posterior factor probabilities $\mathbb{E}[\gamma_j|\text{data}]$ estimated with the BMA approach outlined in Section 2 for factors constructed based on put options with high or low retail activity. At the beginning of each month, we sort puts based on the median of small customer volume over total non-market-maker volume and construct factors and traded option test assets for both subsamples. Traded test assets are based on calls. The sample period is from June 2005 to March 2021. All other specifications are detailed in Figure 2.

Table D1: Cross-sectional pricing performance - Calls, retail split.

This table reports four performance measures of cross-sectional pricing for different factor models. For the BMA-SDF, prices of risk are estimated using the methodology outlined in Section 2. For the benchmark models, we use GMM with a GLS weighting matrix to estimate risk prices. Benchmark models are described in Internet Appendix IA2. *CAPM* refers to a one-factor model utilizing only `ew_ret`, whereas *51 factors* utilizes all 30 traded and 21 non-traded factors. Out-of-sample test assets are the 26 long-short factors detailed in Internet Appendix IA1.4.2 as well as 17 long portfolios based on FF17 industry sorts. At the beginning of each month, we sort puts based on the median of small customer volume over total non-market-maker volume and construct factors and traded option test assets for both subsamples. RMSE and MAPE are based on returns standardized to an annual volatility of 100%.

Panel A: High retail									
BMA-SDF	RMSE	MAPE	R^2_{ols}	R^2_{gls}	Model	RMSE	MAPE	R^2_{ols}	R^2_{gls}
10%-SR_{pr}	1.379	1.041	0.145	0.046	51 factors	1.085	0.83	0.471	-0.735
25%-SR_{pr}	1.088	0.808	0.467	0.198	CAPM	1.486	1.149	0.007	0.016
40%-SR_{pr}	0.9	0.638	0.636	0.279	HVX	1	0.751	0.551	0.285
60%-SR_{pr}	0.759	0.511	0.741	0.347	ZHCT	0.994	0.819	0.555	0.111
75%-SR_{pr}	0.692	0.461	0.785	0.383	AN	1.506	1.208	-0.02	0.095
90%-SR_{pr}	0.639	0.421	0.817	0.403	TW	1.146	0.907	0.409	-0.171
Panel B: Low retail									
BMA-SDF	RMSE	MAPE	R^2_{ols}	R^2_{gls}	Model	RMSE	MAPE	R^2_{ols}	R^2_{gls}
10%-SR_{pr}	1.069	0.825	0.096	-0.098	51 factors	0.657	0.497	0.659	-0.115
25%-SR_{pr}	0.911	0.709	0.344	0.044	CAPM	1.125	0.891	-0.001	-0.075
40%-SR_{pr}	0.78	0.588	0.519	0.127	HVX	0.757	0.584	0.547	0.186
60%-SR_{pr}	0.659	0.471	0.656	0.201	ZHCT	0.892	0.722	0.371	0.062
75%-SR_{pr}	0.598	0.413	0.717	0.245	AN	1.13	0.809	-0.01	-0.338
90%-SR_{pr}	0.556	0.376	0.756	0.281	TW	0.679	0.482	0.636	-0.083

Table D2: Cross-sectional pricing performance - Puts, retail split.
This table reports four performance measures of cross-sectional pricing for different factor models. For the BMA-SDF, prices of risk are estimated using the methodology outlined in Section 2. For the benchmark models, we use GMM with a GLS weighting matrix to estimate risk prices. Benchmark models are described in Internet Appendix IA2. *CAPM* refers to a one-factor model utilizing only `ew_ret`, whereas *51 factors* utilizes all 30 traded and 21 non-traded factors. Out-of-sample test assets are the 26 long-short factors detailed in Internet Appendix IA1.4.2 as well as 17 long portfolios based on FF17 industry sorts. At the beginning of each month, we sort puts based on the median of small customer volume over total non-market-maker volume and construct factors and traded option test assets for both subsamples. RMSE and MAPE are based on returns standardized to an annual volatility of 100%.

Panel A: High retail									
BMA-SDF	RMSE	MAPE	R^2_{ols}	R^2_{gls}	Model	RMSE	MAPE	R^2_{ols}	R^2_{gls}
10%-SR_{pr}	1.5	1.041	0.144	-0.202	51 factors	0.708	0.544	0.81	-0.035
25%-SR_{pr}	1.174	0.85	0.476	0.04	CAPM	1.606	1.173	0.019	-0.12
40%-SR_{pr}	0.954	0.704	0.654	0.156	HVX	1.402	1.062	0.253	0.126
60%-SR_{pr}	0.779	0.588	0.769	0.234	ZHCT	1.006	0.824	0.615	0.214
75%-SR_{pr}	0.691	0.524	0.819	0.275	AN	1.564	1.072	0.071	-0.318
90%-SR_{pr}	0.624	0.478	0.852	0.316	TW	1.334	0.886	0.324	-0.18

Panel B: Low retail									
BMA-SDF	RMSE	MAPE	R^2_{ols}	R^2_{gls}	Model	RMSE	MAPE	R^2_{ols}	R^2_{gls}
10%-SR_{pr}	1.14	0.846	0.095	-0.409	51 factors	1.023	0.849	0.271	-1.007
25%-SR_{pr}	0.97	0.743	0.344	-0.14	CAPM	1.196	0.915	0.003	-0.225
40%-SR_{pr}	0.826	0.641	0.525	-0.005	HVX	1.051	0.833	0.231	0.004
60%-SR_{pr}	0.685	0.53	0.673	0.1	ZHCT	0.92	0.786	0.411	0.198
75%-SR_{pr}	0.608	0.471	0.742	0.155	AN	1.18	0.863	0.03	-1.344
90%-SR_{pr}	0.551	0.426	0.788	0.201	TW	1.064	0.796	0.212	-0.571

Internet Appendix

A Bayesian Stochastic Discount Factor for the Cross-Section of Individual Equity Options

- Appendix IA1 describes the set of traded and non-traded factors in detail.
- Appendix IA2 describes the reduced-form option factor models used throughout the paper.
- Appendix IA3 replicates the main findings with alternative in-sample test assets.
- Appendix IA4 reports the main results when excluding the factors `ivrv` and `omom` from the analysis.
- Appendix IA5 reports the out-of-sample pricing performance separately for three different sets of test assets.
- Appendix IA6 reports the main results when weighting options by their dollar-open interest and by the market capitalization of the options' underlying.
- Appendix IA7 reports the main results when using margin-adjusted option returns.
- Appendix IA8 reports the main results when using a more conservative prior belief on factor inclusion probabilities.

IA1 Traded and non-traded factors

IA1.1. Traded factors

IA1.1.1. Option factors

1. Embedded leverage (`embedlev`): The embedded leverage of the option contract following Frazzini & Pedersen (2022) which has been also used in Büchner and Kelly (2022) for an option factor model for S&P 500 index options.
2. Delta-hedging costs (`hc`): Delta hedging costs are calculated according to Tian & Wu (2023). Specifically, delta-hedging costs, $hc_{t,i}$, at time$-t$ on stock i are given as

   $$hc_{i,t} = \sigma_{t,i}\sqrt{(1-\rho_{t,i}^2)/DV_{t,i}},$$

   where $\sigma_{t,i}$ denotes the stock's historical return volatility estimator, $\rho_{t,i}$ the return correlation of the stock with the aggregate market portfolio, and $DV_{t,i}$ denotes the average dollar trading volume (in thousands) on the stock.
3. Volatility risk (`vr`): Volatility risk, $vr_{t,i}$, is calculated according to Tian & Wu (2023). It is the standard deviation of daily changes of the stock i's 1-month at-the-money option implied volatility over the past month t.
4. Historical jump risk (`jr`): (Historical) jump risk follows Tian & Wu (2023) in its construction. It is the product of stock's excess kurtosis and historical return volatility over the past month.
5. Volatility of implied volatility (`vov`): Volatility of implied volatility is calculated following Ruan (2020) as the standard deviation of 30-day at-the-money volatility scaled by the mean of 30-day at-the-money volatility over the previous month.
6. Option illiquidity (`optspread`): Option illiquidity is measured as the option bid-ask spread following Christoffersen et al. (2018).

7. Option momentum (`omom`): Option momentum as the average stock-level option return over the past year skipping the most recent month as in Heston et al. (2023) and Käfer et al. (2023).
8. Historical stock volatility (`hvol`): The historical volatility of stock returns measured over the past month using daily data as in Hu & Jacobs (2020).
9. Systematic volatility (`sysvol`): The systematic volatility of the underlying stock's returns. Following Aretz et al. (2023), it is estimated as the square root of the annualized variance of the fitted value from a time-series regression of the stock's return on the Fama and French (2018) 6-factor model over the past 24 months.
10. The term structure of implied at-the-money volatility (`ivterm`): The difference between short and long term at-the-money implied volatility following Vasquez (2017). Following the implementation of Goyal & Saretto (2024), we take the difference between 365 and 30-days to expiration at-the-money implied volatility from the implied volatility surface of OptionMetrics. At-the-money implied volatility is the average of the put and call implied volatility with an absolute delta of 0.5.
11. Stock return autocorrelation (`ac`): The autocorrelation of daily returns over the last 6 months requiring at least 100 observations (Jeon et al., 2019).
12. Average of 10 highest past returns (`max10`): As in Byun & Kim (2016), the average of the 10 highest daily returns over the last 3 months following Bali et al. (2011).
13. Default risk (`defrisk`): Following Vasquez & Xiao (2023), we calculate the default probability of the underlying stock as in Bharath and Shumway (2008).
14. Idiosyncratic skewness (`iskew`): The third moment of the residuals from regressing the stock returns on the market return and its square following Byun & Kim (2016).
15. Total skewness (`tskew`): The third moment of the residuals from regressing the stock returns on the market return and its square following Byun & Kim (2016).
16. Idiosyncratic volatility (`ivol`): The idiosyncratic volatility of the underlying with respect to the Fama & French (1993) 3-factor model over the past month as in Cao & Han (2013). The construction follows Goyal & Saretto (2024).

17. Implied volatility minus realized volatility (`ivrv`): The difference between implied and realized volatility as in Goyal & Saretto (2009).

18. Stock illiquidity (`amihud`): Following and Zhan et al. (2022) and Kanne et al. (2023), we include the Amihud (2002) illiquidity measure over the past month.

19. Short interest (`rsi`): The ratio between short interest (taken from Compustat's Supplemental Short Interest File (*shortintadj*)) and the total shares outstanding (Ramachandran & Tayal, 2021).

20. 1-year new stock issues (`issue_1y`): Following Zhan et al. (2022), we include the one-year change in the log of the number of shares outstanding (Pontiff and Woodgate, 2008). The data is taken from Jensen et al. (2023).

21. 5-year new stock issues (`issue_5y`): Following Zhan et al. (2022), we calculate the five-year change in the log of the number of shares outstanding (Daniel and Titman, 2006).

22. Altman Z-score (`zscore`): Following Zhan et al. (2022), we include the Altman Z-score (Dichev, 1998). The data is taken from Jensen et al. (2023).

23. Analyst dispersion (`disp`): Following Zhan et al. (2022), we include analyst earnings forecast dispersion computed as the standard deviation of analysts' annual earnings-per-share forecasts over the absolute value of the average forecast (Diether et al., 2002). The data is constructed using the replication code of Green et al. (2017).[1]

24. Cash-to-assets ratio (`cash_at`): Following Zhan et al. (2022), we include the corporate cash holdings over total assets (Palazzo, 2012). The data is taken from Jensen et al. (2023).

25. Cash flow volatility (`ocfq_saleq_std`): Following Zhan et al. (2022), we include the standard deviation of quarterly reported operating cash flows over quarterly sales h09. The data is taken from Jensen et al. (2023).

26. Operating profits-to-book equity (`ope_be`): The operating profits-to-book equity ratio as in Fama & French (2015). The data is taken from Jensen et al. (2023).

27. Profit margin (`ebit_sale`): Following Zhan et al. (2022), we include the profit margin defined as

[1] `https://sites.google.com/site/jeremiahrgreenacctg/home`.

EBIT over total sales (Soliman, 2008). The data is taken from Jensen et al. (2023).

28. Net total issuance (`netis_at`): Net total issuance defined as total share and debt issuance minus cash dividend payments as in Bradshaw et al. (2006). The data is taken from Jensen et al. (2023).
29. Stock price (`log_price`): Following Zhan et al. (2022) and Boulatov et al. (2022), we take the log of the underlying stock's close price. The data is directly taken from CRSP.
30. The equal-weighted return of all option contracts (`ew_ret`): As in Horenstein et al. (2022), we include an equal-weighted return of all option contracts which is calculated as the mean of across all decile portfolios across all above option factors.

IA1.1.2. Stock factors

1. Market risk premium (`Mkt-RF`): The market return in excess of the risk-free rate of the capital asset pricing model (Sharpe, 1964; Lintner, 1965). The data is taken from Kenneth French's website.
2. Stock value (`SMB`): The stock size factor following Fama & French (1992) and Fama & French (1993) defined as the average return on the three (value, neutral, and growth) small portfolios minus the average return on the three big portfolios. The data is taken from Kenneth French's website.
3. Stock size (`HML`): The stock value factor following Fama & French (1992) and Fama & French (1993) defined as the average return on the two (small and big) value, measured by the book-to-market ratio, portfolios minus the average return on the two growth portfolios. The data is taken from Kenneth French's website.
4. Stock profitability (`RMW`): The stock profitability factor following Fama & French (2015) defined as the average return on the two (small and big) robust operating profitability portfolios minus the average return on the two weak operating profitability portfolios. The data is taken from Kenneth French's website.
5. Stock investment (`CMA`): The stock investment factor following Fama & French (2015) defined as the average return on the two (small and big) conservative investment portfolios minus the average return on the two aggressive investment portfolios. The data is taken from Kenneth French's website.

6. Stock momentum (`Mom`): The Carhart (1997) stock momentum factor following Fama & French (2015) defined as the average return on the two (small and big) high prior return portfolios minus the average return on the two low prior return portfolios. Prior returns are computed over the months $t-2$ to $t-12$. The data is taken from Kenneth French's website.

IA1.2. Non-traded factors

1. Intermediary capital nontraded risk (`CPTL`): The intermediary capital nontraded risk factor of He et al. (2017). The data is downloaded from Zhiguo He's website at `https://voices.uchicago.edu/zhiguohe/`.
2. Economic policy uncertainty (`EPU`): The first difference in the economic policy uncertainty index. The data is taken from FRED.
3. Macroeconomic uncertainty (`UNC`): The first difference in the macroeconomic uncertainty index lagged by one month to align the forecast to the returns observed in month t. The data is taken from Sydney Ludvigson's website at `https://www.sydneyludvigson.com/`.
4. Financial economic uncertainty (`UNCf`): The first difference in the financial economic uncertainty index lagged by one month to align the forecast to the returns observed in month t. The data is taken from Sydney Ludvigson's website at `https://www.sydneyludvigson.com/`.
5. Real economic uncertainty (`UNCr`): The first difference in the real economic uncertainty index lagged by one month to align the forecast to the returns observed in month t. Data is taken from Sydney Ludvigson's website at `https://www.sydneyludvigson.com/`.
6. Volatility risk, VIX (`VIX`): The first difference in the CBOE VIX index. The data is downloaded from `https://www.cboe.com/tradable_products/vix/vix_historical_data/`.
7. Volatility-of-volatility risk, VIXVOL (`VIXVOL`): A range-based measure of the volatility of aggregate volatility based on daily readings of the VIX index. The construction follows Agarwal et al. (2017).
8. Market tail risk (`SKEW`): The first difference in the CBOE SKEW index which estimates market tail

risk. The data is downloaded from `https://www.cboe.com/us/indices/dashboard/skew/`.

9. SKEW term structure (`SKEWTS`): The first difference in the CBOE SKEW term structure. It is the difference between 182d and 30d SKEW. Data downloaded from `https://www.cboe.com/us/indices/dashboard/skew/`.
10. Correlation risk (`ICRC`): The payoff of monthly S&P 500 correlation swaps which is the difference between the implied and realized correlation (Buraschi, Kosowski, & Trojani, 2014).
11. Aggregate liquidity risk (`LIQ`): The aggregate liquidity innovation factor of Pástor and Stambaugh (2003). The data is downloaded from Robert Stambaugh's website at `https://finance.wharton.upenn.edu/~stambaug/`.
12. Sentiment (`SENT`): The sentiment index of Baker and Wurgler (2006) orthogonalized with respect to macroeconomic variables. The data is obtained from Jeffrey Wurgler's website at `https://pages.stern.nyu.edu/~jwurgler/`.
13. Interest rate term structure (`TERM`): The slope of the term structure of interest rates. TERM is calculated as the difference between U.S. Treasury Securities at 10-year constant maturity and 3-month Treasury bill secondary market rates. The data is obtained from FRED.
14. Default spread (`DEF`): The default spread defined as the yield difference between Moody's AAA and BAA corporate bond yields. The data is taken from FRED.
15. Financial Stress (`STLFSI`): The St. Louis Fed Financial Stress Index. Data is obtained from FRED.

IA1.3. In-sample test assets

1. The long-short portfolios for all traded *option* factors described in Appendix IA1.1.1.
2. 5×5 long-only portfolios independently double-sorted on implied minus realized volatility (`ivrv`) and dollar open interest (`doi`).

IA1.4. Out-of-sample test assets

IA1.4.1. 17 Fama-French industry portfolios (FF17)

We construct long-only industry portfolios using the Fama-French 17 industry classification. The industry definitions based on Compustat SIC codes are available on Kenneth French's website. The coverage of the industry return set is 100% for calls and 99.9% for puts. 6 missing monthly returns for industry 10, fabricated products, are set to 0.

IA1.4.2. Remaining option anomalies proposed by Goyal & Saretto (2024)

We construct long-short portfolios for each of the option anomalies proposed by Goyal & Saretto (2024) which are not part of our set of traded factors in Appendix IA1.1.1:

1. At-the-money implied volatility (`atmiv`): The average of put and call implied volatility with an absolute delta of 0.5. The implied volatilities are extracted from the 30-day implied volatility surface of OptionMetrics.
2. Debt to total assets (`debt_at`): The firm's leverage defined as total debt over total assets. The data is taken from Jensen et al. (2023).
3. Market capitalization (`mcap`): The total market value of equity. The data is taken from Jensen et al. (2023).
4. Financial debt (`debt`): The firm's total book value of debt. The data is taken from Jensen et al. (2023) using debt to total assets (`debt_at`) and total assets (`assets`).
5. Net equity issuance (`eqnetis_at`): Net equity issuance defined as total share issuance minus cash dividend payments (Bradshaw et al., 2006). The data is taken from Jensen et al. (2023).
6. Risk-neutral volatility (`rniv`): The model-free implied volatility. We use the 30-day implied volatility surface of OptionMetrics. The code is taken from Grigory Vilkov's website.[2]

[2] `https://www.vilkov.net/index.html`.

7. Realized skewness (`skew`): The skewness of daily log returns over the last 12 months requiring at least 150 observations.
8. Risk-neutral skewness (`rns`): The model-free implied skewness constructed from 30 days out-of-the-money call and out-of-the-money put option prices as in Bakshi & Kapadia (2003). We use the 30-day implied volatility surface of OptionMetrics. The code is taken from Grigory Vilkov's website.
9. Realized skewness minus risk-neutral skewness (`diff_skew`): The difference between `skew` and `rns`.
10. Realized kurtosis (`kurtosis`): The kurtosis of daily log returns over the last 12 months requiring at least 150 observations.
11. Risk-neutral kurtosis (`rnk`): The model-free implied kurtosis constructed from 30 days out-of-the-money call and out-of-the-money put option prices as in Bakshi & Kapadia (2003). We use the 30-day implied volatility surface of OptionMetrics. The code is taken from Grigory Vilkov's website.
12. Realized kurtosis minus risk-neutral kurtosis (`diff_kurt`): The difference between `kurtosis` and `rnk`.
13. Share turnover (`turnover_126d`): The total share turnover rate (trading volume over shares outstanding) over the past 126 trading days (6 months) (Datar et al., 1998). The data is taken from Jensen et al. (2023).
14. Total assets (`assets`): The firm's total book value of assets. The data is taken from Jensen et al. (2023).
15. Institutional ownership (`inst`): The institutional ownership in percentage derived from Thomson Reuters 13f holdings.
16. Short-term stock return reversal (`str`): Short-term stock return reversal measured as the return in month $t-1$ (Jegadeesh, 1990). The data is taken from Jensen et al. (2023).
17. Stock return momentum (`mom`): Stock price momentum measured as return from month $t-12$ to $t-1$ (Jegadeesh and Titman, 1993). The data is taken from Jensen et al. (2023).
18. Book to market equity (`be_me`): The book-to-market ratio computed as the book value of equity over the current market value of equity. The data is taken from Jensen et al. (2023).
19. Profitability (`gp_at`): The ratio of gross profits to total assets (Novy-Marx, 2013). The data is taken

from Jensen et al. (2023).

20. Option dollar volume (`odvol`): The option volume multiplied by the option's mid price. The data is directly taken from OptionMetrics.
21. Option dollar open interest (`doi`): The option open interest (number of contracts) multiplied by the option's mid price. The data is directly taken from OptionMetrics.
22. Option contract's implied volatility (`iv`): The implied volatility of the option contract. The data is directly taken from OptionMetrics.
23. Implied volatility slope (`ivslope`): The difference between the implied volatilities of out-of-the-money puts and at-the-money calls (Xing et al., 2010). The data is directly taken from OptionMetrics.
24. Moneyness (`moneyness`): The moneyness of the option contract defined as the strike-to-spot ratio. The data is directly taken from OptionMetrics and CRSP.
25. Option contract's mid price (`mid`): The mid price of the option contract. The data is directly taken from OptionMetrics.
26. Option delta (`delta`): The delta of the option contract. The data is directly taken from OptionMetrics.

IA2 Benchmark factor models

We consider five benchmark option factor models. Each model is separately constructed for call and put options using our daily delta-hedging return definition and option filters detailed in Section 3.

Option market portfolio, CAPM: Analogous to the equity market, we construct an option market portfolio and use it for a one-factor model. We approximate the market factor using `ew_ret`.

Horenstein et al. (2022), HVX: Horenstein et al. (2022) adopt the methodology of Lettau and Pelger (2020) and apply it to single-name options. Their identified latent factors are captured by three tradable option factors: an equal-weighted option portfolio, the long-short factor based on the difference in implied and realized volatility, and the long-short factor based on the volatility of implied volatility. We approximate their factors using `ew_ret`, `ivrv`, and `vov`.

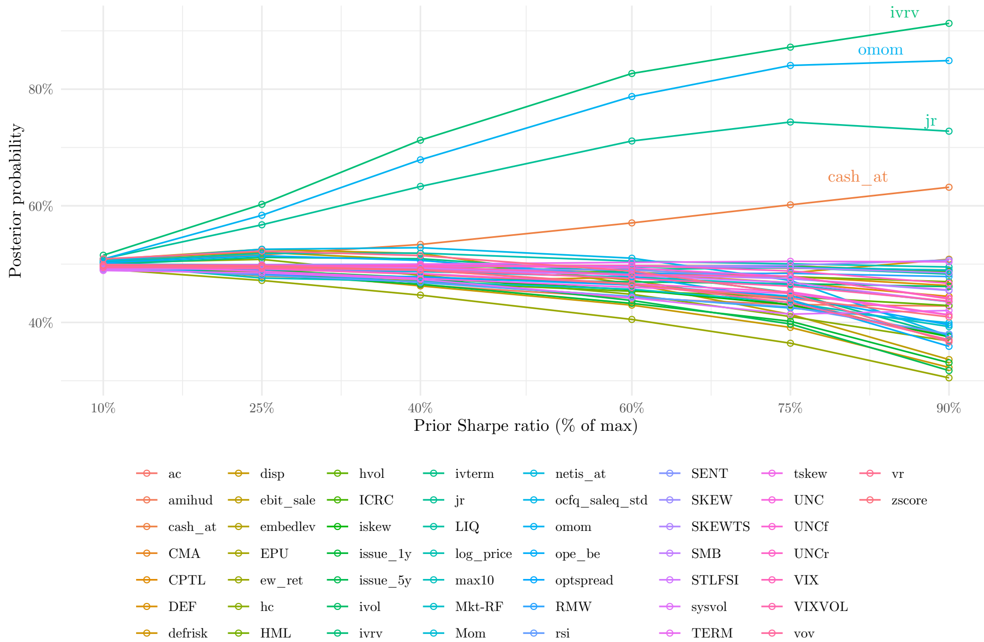
Zhan et al. (2022), ZHCT: Zhan et al. (2022) show that two factors, the underlying's illiquidity and idiosyncratic volatility, can price 9 out of 10 stock-based characteristics in the options market. We approximate their factors using `amihud`, and `ivol`.

Tian & Wu (2023), TW: Tian & Wu (2023) propose a five-factor model to price the cross-section of option returns. The first three factors are deemed as primary risks of option market makers: delta-hedging costs, volatility risk, and jump risk. These three factors are augmented by a historical risk premium (captured by option momentum) and a volatility risk premium (captured by the difference between implied and realized volatility). We approximate their factors using `hc`, `vr`, `jr`, `omon`, and `ivrv`.

Agarwal & Naik (2004), AN: Agarwal & Naik (2004) construct S&P 500 index option-based risk factors using at-the-money and out-of-the-money option contracts for calls and puts, respectively. Although the option-based risk factors are constructed using index options, they have also been frequently used for single-name options. Hence, we include them as a benchmark model.

IA3 Alternative in-sample test assets

(a) Calls



(b) Puts

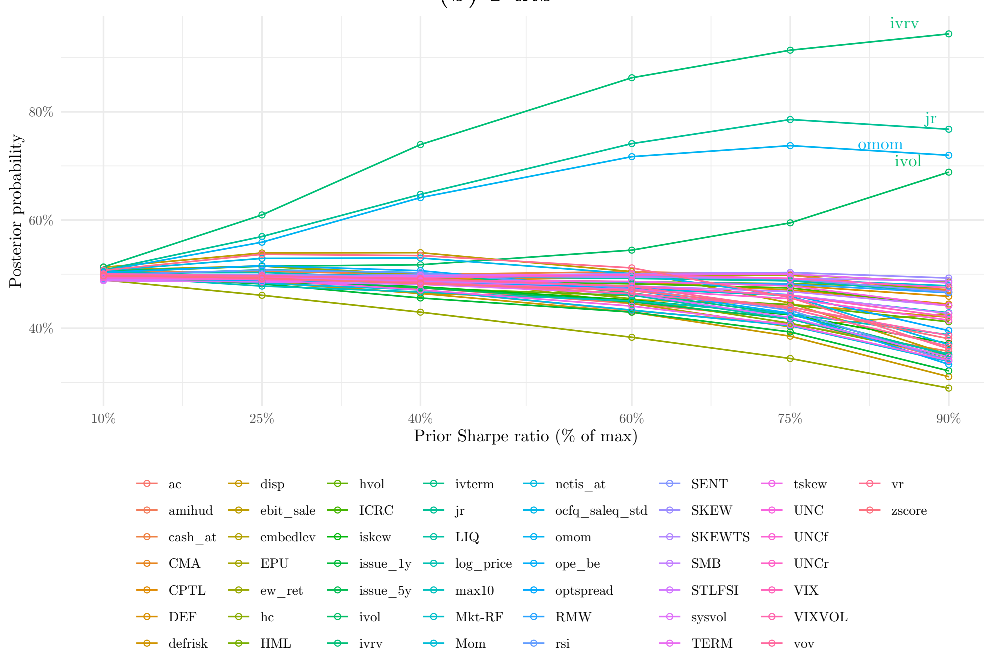


Fig. IA3.1. Posterior factor inclusion probabilities for alternative in-sample test assets.

Notes: This figure shows posterior factor probabilities $\mathbb{E}[\gamma_j|\text{data}]$ estimated with the BMA approach outlined in Section 2. We replace the 25 portfolios based on double sorts on `ivrv` and `doi` with 25 portfolios based on double sorts on `be_me` and `mcap`. Results for calls are shown in (a) and for puts in (b). All other specifications are detailed in Figure 1.

Table IA3.1: Cross-sectional pricing performance - Calls, alternative in-sample test assets.

This table reports four performance measures of cross-sectional pricing for different factor models. For the BMA-SDF, prices of risk are estimated using the methodology outlined in Section 2, but replacing the 25 in-sample test assets based on double sorts on `ivrv` and `doi` with 25 portfolios based on double sorts on `be_me` and `mcap`. For the benchmark models, we use GMM with a GLS weighting matrix to estimate risk prices. Benchmark models are described in Appendix IA2. *CAPM* refers to a one-factor model utilizing only `ew_ret`, whereas *51 factors* utilizes all 30 traded and 21 non-traded factors. Out-of-sample test assets are the 26 long-short factors detailed in Appendix IA1.4.2 as well as 17 long portfolios based on FF17 industry sorts. Portfolio returns are calculated with equal call option weighting. RMSE and MAPE are based on returns standardized to an annual volatility of 100%.

Panel A: In-Sample Pricing, Test Assets: 51 Factors and 25 IVRV-DOI portfolios									
BMA-SDF	RMSE	MAPE	R^2_{ols}	R^2_{gls}	Model	RMSE	MAPE	R^2_{ols}	R^2_{gls}
10%-SR_{pr}	1.232	0.829	0.132	0.05	51 factors	0.031	0.018	0.999	0.989
25%-SR_{pr}	0.981	0.656	0.417	0.159	CAPM	1.304	0.914	0.013	0.015
40%-SR_{pr}	0.788	0.521	0.623	0.282	HVX	0.978	0.692	0.442	0.29
60%-SR_{pr}	0.563	0.364	0.808	0.439	ZHCT	1.07	0.813	0.366	0.074
75%-SR_{pr}	0.422	0.284	0.892	0.545	AN	1.179	0.91	0.159	0.081
90%-SR_{pr}	0.298	0.226	0.946	0.653	TW	1.137	0.909	0.425	0.415
Panel B: Out-of-Sample Pricing, Test Assets: 26 Factors and 17 Industry Portfolios									
BMA-SDF	RMSE	MAPE	R^2_{ols}	R^2_{gls}	Model	RMSE	MAPE	R^2_{ols}	R^2_{gls}
10%-SR_{pr}	1.407	0.933	0.153	0.008	51 factors	0.756	0.521	0.755	-0.141
25%-SR_{pr}	1.133	0.772	0.45	0.137	CAPM	1.486	1.015	0.054	-0.005
40%-SR_{pr}	0.95	0.639	0.613	0.227	HVX	1.025	0.863	0.55	0.303
60%-SR_{pr}	0.781	0.502	0.739	0.318	ZHCT	1.074	0.824	0.506	0.041
75%-SR_{pr}	0.69	0.427	0.796	0.366	AN	1.327	1.078	0.246	0.083
90%-SR_{pr}	0.611	0.38	0.84	0.411	TW	1.147	0.872	0.437	0.02

Table IA3.2: Cross-sectional pricing performance - Puts, alternative in-sample test assets.

This table reports four performance measures of cross-sectional pricing for different factor models. For the BMA-SDF, prices of risk are estimated using the methodology outlined in Section 2, but replacing the 25 in-sample test assets based on double sorts on `ivrv` and `doi` with 25 portfolios based on double sorts on `be_me` and `mcap`. For the benchmark models, we use GMM with a GLS weighting matrix to estimate risk prices. Benchmark models are described in Appendix IA2. *CAPM* refers to a one-factor model utilizing only `ew_ret`, whereas *51 factors* utilizes all 30 traded and 21 non-traded factors. Out-of-sample test assets are the 26 long-short factors detailed in Appendix IA1.4.2 as well as 17 long portfolios based on FF17 industry sorts. Portfolio returns are calculated with equal put option weighting. RMSE and MAPE are based on returns standardized to an annual volatility of 100%.

Panel A: In-Sample Pricing, Test Assets: 51 Factors and 25 IVRV-DOI portfolios

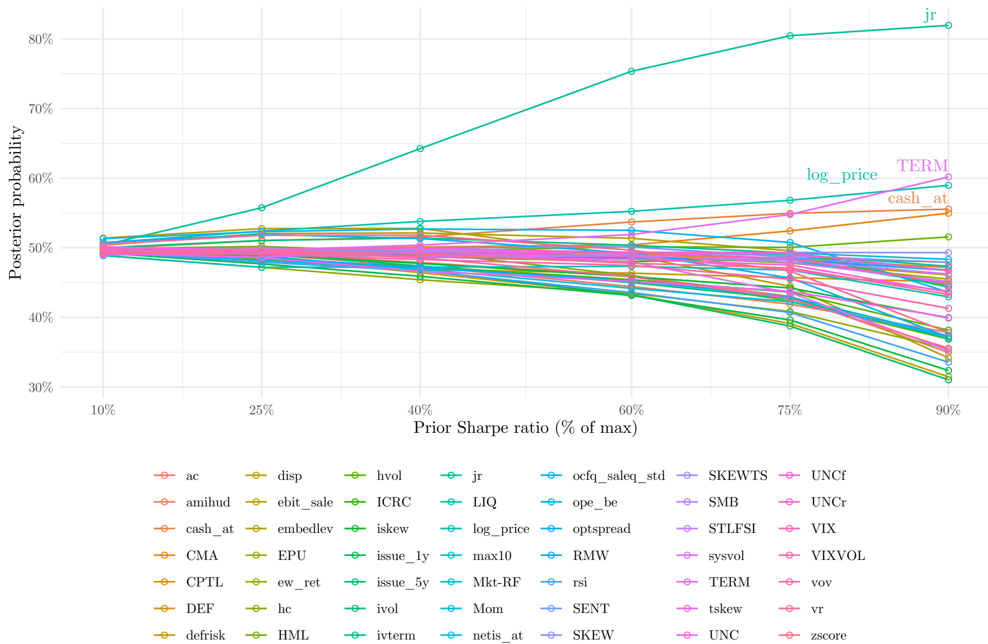
BMA-SDF	RMSE	MAPE	R^2_{ols}	R^2_{gls}	Model	RMSE	MAPE	R^2_{ols}	R^2_{gls}
10%-SR_{pr}	1.428	1.033	0.135	0.045	51 factors	0.038	0.021	0.999	0.988
25%-SR_{pr}	1.103	0.738	0.432	0.163	CAPM	1.479	1.086	-0.006	0.009
40%-SR_{pr}	0.87	0.563	0.636	0.288	HVX	1.361	0.993	0.151	0.259
60%-SR_{pr}	0.621	0.406	0.815	0.441	ZHCT	1.077	0.736	0.444	0.11
75%-SR_{pr}	0.469	0.324	0.896	0.536	AN	1.483	1.086	0.042	0.077
90%-SR_{pr}	0.343	0.257	0.946	0.625	TW	1.366	1.099	0.369	0.387

Panel B: Out-of-Sample Pricing, Test Assets: 26 Factors and 17 Industry Portfolios

BMA-SDF	RMSE	MAPE	R^2_{ols}	R^2_{gls}	Model	RMSE	MAPE	R^2_{ols}	R^2_{gls}
10%-SR_{pr}	1.324	0.919	0.171	-0.27	51 factors	0.979	0.744	0.547	-1.551
25%-SR_{pr}	1.016	0.741	0.512	0.013	CAPM	1.403	1	0.069	-0.135
40%-SR_{pr}	0.808	0.611	0.691	0.179	HVX	1.371	1.115	0.111	0.236
60%-SR_{pr}	0.614	0.462	0.822	0.326	ZHCT	0.885	0.733	0.629	0.119
75%-SR_{pr}	0.508	0.38	0.878	0.406	AN	1.442	1.069	0.017	-0.091
90%-SR_{pr}	0.425	0.32	0.914	0.474	TW	1.225	0.933	0.29	-0.108

IA4 Estimation without ivrv and omom

(a) Calls



(b) Puts

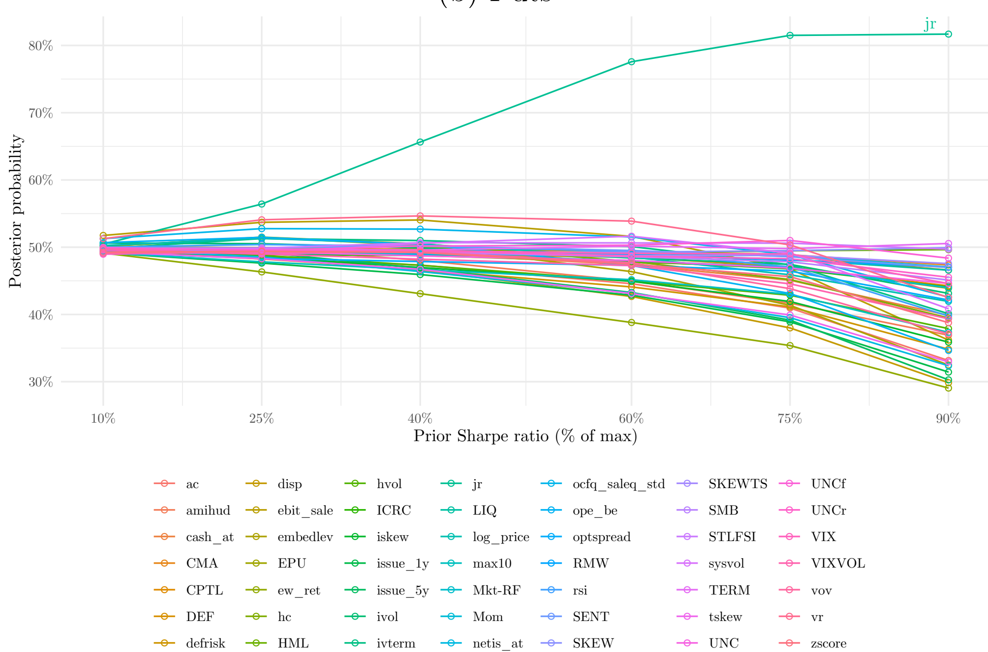


Fig. IA4.1. Posterior factor inclusion probabilities excluding ivrv and omom.

Notes: This figure shows posterior factor probabilities $\mathbb{E}[\gamma_j|\text{data}]$ estimated with the BMA approach outlined in Section 2. The factors based on the spread between option-implied and realized volatility (*ivrv*) as well as on the options' past returns (*omom*) are not considered in the factor set or as test assets. Results for calls are shown in (a) and for puts in (b). All other specifications are detailed in Figure 1.

IA5 Additional out-of-sample test assets

Table IA5.1: Out-of-sample pricing performance - Calls, additional test assets. This table reports four performance measures of cross-sectional out-of-sample pricing for different factor models and three different sets of test assets. For the BMA-SDF, prices of risk are estimated using the methodology outlined in Section 2 and reported in Table A1. For the benchmark models, we use GMM with a GLS weighting matrix to estimate risk prices. Benchmark models are described in Appendix IA2. *CAPM* refers to a one-factor model utilizing only `ew_ret`, whereas *51 factors* utilizes all 30 traded and 21 non-traded factors. In Panel A, the out-of-sample test assets are 17 long portfolios based on FF17 industry sorts. In Panel B, the test assets are the 26 long-short factors detailed in Appendix IA1.4.2. In Panel C, the test assets are 25 long portfolios based on independent 5×5 portfolio sorts on the book-to-market ratio (`be_me`) and the market capitalization (`mcap`) of the options' underlying stocks. Portfolio returns are calculated with equal call option weighting. RMSE and MAPE are based on returns standardized to an annual volatility of 100%.

BMA-SDF	RMSE	MAPE	R^2_{ols}	R^2_{gls}	Model	RMSE	MAPE	R^2_{ols}	R^2_{gls}
Panel A: Out-of-Sample Pricing, Test Assets: 17 Industry Portfolios									
10%-SR_{pr}	0.359	0.222	0.101	0.077	51 factors	0.936	0.795	-5.113	-4.921
25%-SR_{pr}	0.309	0.205	0.331	0.258	CAPM	0.375	0.229	0.018	0.011
40%-SR_{pr}	0.263	0.19	0.517	0.401	HVX	0.382	0.271	-0.018	-0.085
60%-SR_{pr}	0.216	0.168	0.675	0.51	ZHCT	0.358	0.253	0.104	0.129
75%-SR_{pr}	0.2	0.16	0.722	0.533	AN	0.406	0.306	-0.153	-0.234
90%-SR_{pr}	0.2	0.158	0.719	0.516	TW	0.391	0.304	-0.065	-0.062
Panel B: Out-of-Sample Pricing, Test Assets: 26 Factors									
10%-SR_{pr}	1.786	1.404	0.151	0.012	51 factors	1.741	1.328	0.194	-1.851
25%-SR_{pr}	1.428	1.107	0.458	0.144	CAPM	1.893	1.51	0.047	-0.037
40%-SR_{pr}	1.184	0.897	0.627	0.263	HVX	1.272	1.164	0.57	0.248
60%-SR_{pr}	0.972	0.695	0.749	0.39	ZHCT	1.294	1.014	0.555	0.022
75%-SR_{pr}	0.862	0.593	0.802	0.462	AN	1.533	1.344	0.375	0.104
90%-SR_{pr}	0.78	0.528	0.838	0.521	TW	1.45	1.26	0.441	0.099
Panel C: Out-of-Sample Pricing, Test Assets: 25 BM-MCAP portfolios									
10%-SR_{pr}	0.388	0.272	0.163	0.103	51 factors	0.587	0.497	-0.914	-5.73
25%-SR_{pr}	0.291	0.182	0.531	0.316	CAPM	0.43	0.31	-0.025	0.012
40%-SR_{pr}	0.216	0.134	0.741	0.492	HVX	0.373	0.304	0.227	0.307
60%-SR_{pr}	0.163	0.11	0.852	0.625	ZHCT	0.29	0.193	0.534	0.153
75%-SR_{pr}	0.149	0.108	0.877	0.662	AN	0.4	0.287	0.112	-0.06
90%-SR_{pr}	0.147	0.117	0.88	0.661	TW	0.381	0.307	0.196	0.337

Table IA5.2: Out-of-sample pricing performance - Puts, additional test assets. This table reports four performance measures of cross-sectional out-of-sample pricing for different factor models and three different sets of test assets. For the BMA-SDF, prices of risk are estimated using the methodology outlined in Section 2 and reported in Table A2. For the benchmark models, we use GMM with a GLS weighting matrix to estimate risk prices. Benchmark models are described in Appendix IA2. *CAPM* refers to a one-factor model utilizing only `ew_ret`, whereas *51 factors* utilizes all 30 traded and 21 non-traded factors. In Panel A, the out-of-sample test assets are 17 long portfolios based on FF17 industry sorts. In Panel B, the test assets are the 26 long-short factors detailed in Appendix IA1.4.2. In Panel C, the test assets are 25 long portfolios based on independent 5×5 portfolio sorts on the book-to-market ratio (`be_me`) and the market capitalization (`mcap`) of the options' underlying stocks. Portfolio returns are calculated with equal put option weighting. RMSE and MAPE are based on returns standardized to an annual volatility of 100%.

BMA-SDF	RMSE	MAPE	R^2_{ols}	R^2_{gls}	Model	RMSE	MAPE	R^2_{ols}	R^2_{gls}
Panel A: Out-of-Sample Pricing, Test Assets: 17 Industry Portfolios									
10%-SR_{pr}	0.364	0.246	0.063	0.099	51 factors	0.824	0.652	-3.805	-4.9
25%-SR_{pr}	0.332	0.237	0.22	0.255	CAPM	0.37	0.246	0.03	0.037
40%-SR_{pr}	0.293	0.219	0.392	0.411	HVX	0.428	0.297	-0.295	-0.105
60%-SR_{pr}	0.245	0.191	0.577	0.557	ZHCT	0.415	0.296	-0.219	0.027
75%-SR_{pr}	0.221	0.171	0.654	0.609	AN	0.429	0.321	-0.3	-0.161
90%-SR_{pr}	0.209	0.165	0.691	0.633	TW	0.434	0.321	-0.334	-0.035
Panel B: Out-of-Sample Pricing, Test Assets: 26 Factors									
10%-SR_{pr}	1.678	1.352	0.173	-0.264	51 factors	1.852	1.549	-0.007	-2.471
25%-SR_{pr}	1.267	1.036	0.529	-0.059	CAPM	1.76	1.422	0.09	-0.314
40%-SR_{pr}	0.983	0.816	0.716	0.121	HVX	1.695	1.484	0.156	0.029
60%-SR_{pr}	0.735	0.605	0.841	0.308	ZHCT	0.978	0.829	0.719	-0.176
75%-SR_{pr}	0.606	0.488	0.892	0.413	AN	1.773	1.431	0.077	-0.096
90%-SR_{pr}	0.513	0.413	0.923	0.497	TW	1.537	1.337	0.306	-0.046
Panel C: Out-of-Sample Pricing, Test Assets: 25 BM-MCAP portfolios									
10%-SR_{pr}	0.441	0.311	0.152	0.136	51 factors	1.007	0.853	-3.426	-11.584
25%-SR_{pr}	0.339	0.224	0.499	0.299	CAPM	0.495	0.359	-0.07	0.023
40%-SR_{pr}	0.264	0.183	0.696	0.427	HVX	0.52	0.428	-0.178	0.041
60%-SR_{pr}	0.209	0.156	0.809	0.511	ZHCT	0.309	0.225	0.582	0.208
75%-SR_{pr}	0.192	0.147	0.839	0.519	AN	0.503	0.38	-0.103	0.064
90%-SR_{pr}	0.189	0.147	0.844	0.49	TW	0.44	0.357	0.155	0.185

IA6 Value-weighted returns

In the main analyses, we consider equal-weighted portfolios. In the following, we consider two alternative portfolio weighting schemes. In Figure IA6.1 and Tables IA6.1 and IA6.2, we weigh options by their respective dollar-open interest. Similar to Jensen et al. (2023), we limit the impact of options with extremely high dollar-open interest by winsorizing (capping) the dollar-open interest at the monthly 80% percentile. In doing so, the weight on options with very little open interest remains minimal, while no single option can dominate the long or short legs of factors. In Figure IA6.2 and Tables IA6.3 and IA6.4, we use the stock market capitalization of the options' underlying for weighting, also winsorized at the monthly 80% percentile.

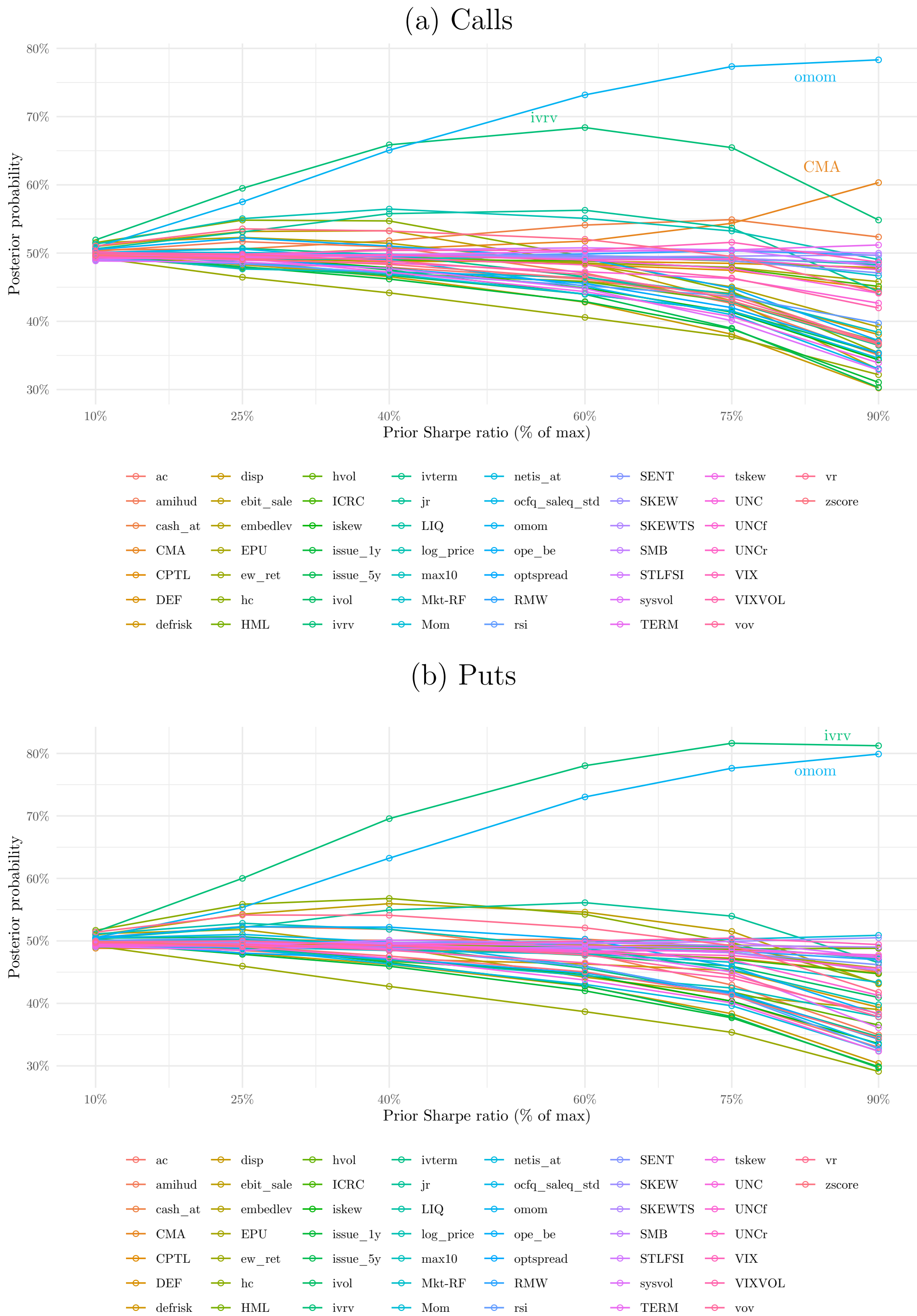


Fig. IA6.1. Posterior factor inclusion probabilities with dollar-open interest weighting.

Notes: Notes: This figure shows posterior factor probabilities $\mathbb{E}[\gamma_j|\text{data}]$ estimated with the BMA approach outlined in Section 2. Results for calls are shown in (a) and for puts in (b). Portfolios are constructed by weighting options by their capped dollar-open interest. All other specifications are detailed in Figure 1.

Table IA6.1: Cross-sectional pricing performance - Calls, dollar-open interest weighting.

This table reports four performance measures of cross-sectional pricing for different factor models. For the BMA-SDF, prices of risk are estimated using the methodology outlined in Section 2. For the benchmark models, we use GMM with a GLS weighting matrix to estimate risk prices. Benchmark models are described in Appendix IA2. *CAPM* refers to a one-factor model utilizing only `ew_ret`, whereas *51 factors* utilizes all 30 traded and 21 non-traded factors. Out-of-sample test assets are the 26 long-short factors detailed in Appendix IA1.4.2 as well as 17 long portfolios based on FF17 industry sorts. Portfolio returns are calculated by weighting call options by their capped dollar-open interest. RMSE and MAPE are based on returns standardized to an annual volatility of 100%.

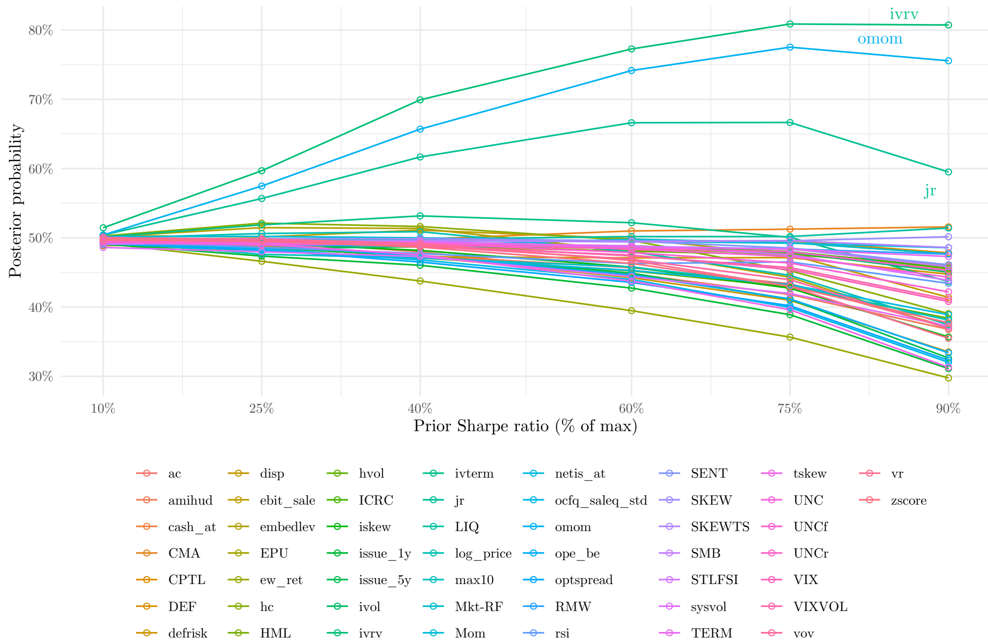
Panel A: In-Sample Pricing, Test Assets: 51 Factors and 25 IVRV-DOI portfolios									
BMA-SDF	RMSE	MAPE	R^2_{ols}	R^2_{gls}	Model	RMSE	MAPE	R^2_{ols}	R^2_{gls}
10%-SR_{pr}	0.975	0.78	0.097	0.034	51 factors	0.115	0.064	0.988	0.884
25%-SR_{pr}	0.806	0.64	0.349	0.103	CAPM	1.003	0.852	-0.023	0.021
40%-SR_{pr}	0.684	0.54	0.524	0.169	HVX	0.571	0.467	0.677	0.151
60%-SR_{pr}	0.578	0.455	0.662	0.248	ZHCT	0.736	0.57	0.453	0.075
75%-SR_{pr}	0.522	0.409	0.727	0.309	AN	0.941	0.823	0.114	0.128
90%-SR_{pr}	0.471	0.37	0.779	0.383	TW	0.539	0.389	0.754	0.198
Panel B: Out-of-Sample Pricing, Test Assets: 26 Factors and 17 Industry Portfolios									
BMA-SDF	RMSE	MAPE	R^2_{ols}	R^2_{gls}	Model	RMSE	MAPE	R^2_{ols}	R^2_{gls}
10%-SR_{pr}	0.897	0.649	0.134	-0.147	51 factors	0.935	0.738	0.057	-2.134
25%-SR_{pr}	0.718	0.52	0.445	0.095	CAPM	0.881	0.643	0.163	0.004
40%-SR_{pr}	0.594	0.439	0.62	0.242	HVX	0.503	0.378	0.728	0.368
60%-SR_{pr}	0.499	0.379	0.732	0.359	ZHCT	0.578	0.433	0.64	0.145
75%-SR_{pr}	0.453	0.35	0.779	0.417	AN	0.829	0.646	0.26	-0.035
90%-SR_{pr}	0.422	0.331	0.808	0.451	TW	0.503	0.393	0.728	0.141

Table IA6.2: Cross-sectional pricing performance - Puts, dollar-open interest weighting.

This table reports four performance measures of cross-sectional pricing for different factor models. For the BMA-SDF, prices of risk are estimated using the methodology outlined in Section 2. For the benchmark models, we use GMM with a GLS weighting matrix to estimate risk prices. Benchmark models are described in Appendix IA2. *CAPM* refers to a one-factor model utilizing only `ew_ret`, whereas *51 factors* utilizes all 30 traded and 21 non-traded factors. Out-of-sample test assets are the 26 long-short factors detailed in Appendix IA1.4.2 as well as 17 long portfolios based on FF17 industry sorts. Portfolio returns are calculated by weighting put options by their capped dollar-open interest. RMSE and MAPE are based on returns standardized to an annual volatility of 100%.

Panel A: In-Sample Pricing, Test Assets: 51 Factors and 25 IVRV-DOI portfolios									
BMA-SDF	RMSE	MAPE	R^2_{ols}	R^2_{gls}	Model	RMSE	MAPE	R^2_{ols}	R^2_{gls}
10%-SR_{pr}	1.042	0.833	0.107	0.03	51 factors	0.079	0.044	0.995	0.968
25%-SR_{pr}	0.831	0.661	0.372	0.101	CAPM	1.021	0.899	-0.014	0.021
40%-SR_{pr}	0.696	0.556	0.538	0.157	HVX	0.733	0.612	0.475	0.104
60%-SR_{pr}	0.592	0.466	0.66	0.213	ZHCT	0.643	0.482	0.597	0.112
75%-SR_{pr}	0.541	0.421	0.714	0.252	AN	0.991	0.807	0.126	0.121
90%-SR_{pr}	0.504	0.389	0.752	0.297	TW	0.612	0.444	0.712	0.197
Panel B: Out-of-Sample Pricing, Test Assets: 26 Factors and 17 Industry Portfolios									
BMA-SDF	RMSE	MAPE	R^2_{ols}	R^2_{gls}	Model	RMSE	MAPE	R^2_{ols}	R^2_{gls}
10%-SR_{pr}	0.92	0.681	0.125	-0.203	51 factors	2.62	2.088	-6.098	-21.276
25%-SR_{pr}	0.746	0.56	0.425	0.079	CAPM	0.884	0.648	0.191	0.012
40%-SR_{pr}	0.626	0.473	0.595	0.24	HVX	0.7	0.518	0.493	0.268
60%-SR_{pr}	0.533	0.404	0.707	0.356	ZHCT	0.563	0.438	0.672	0.187
75%-SR_{pr}	0.491	0.374	0.751	0.411	AN	0.941	0.701	0.085	-0.023
90%-SR_{pr}	0.47	0.361	0.772	0.449	TW	0.587	0.457	0.644	0.22

(a) Calls



(b) Puts

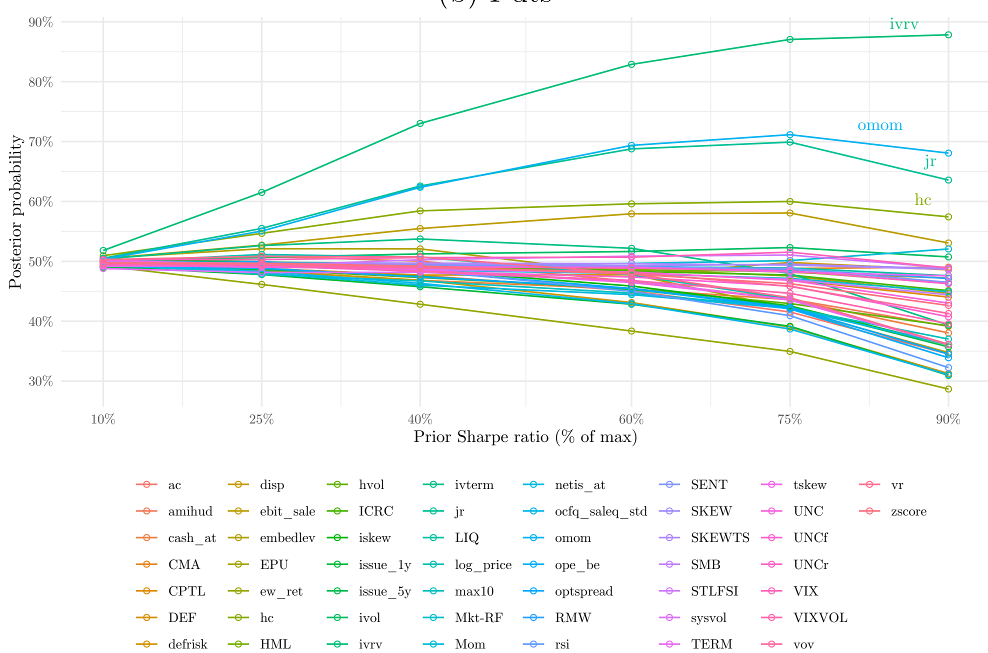


Fig. IA6.2. Posterior factor inclusion probabilities with market-capitalization weighting.

Notes: This figure shows posterior factor probabilities $\mathbb{E}[\gamma_j|\text{data}]$ estimated with the BMA approach outlined in Section 2. Results for calls are shown in (a) and for puts in (b). Portfolios are constructed by weighting options by the capped market capitalization of their underlying stocks. All other specifications are detailed in Figure 1.

Table IA6.3: Cross-sectional pricing performance - Calls, market capitalization weighting.

This table reports four performance measures of cross-sectional pricing for different factor models. For the BMA-SDF, prices of risk are estimated using the methodology outlined in Section 2. For the benchmark models, we use GMM with a GLS weighting matrix to estimate risk prices. Benchmark models are described in Appendix IA2. *CAPM* refers to a one-factor model utilizing only `ew_ret`, whereas *51 factors* utilizes all 30 traded and 21 non-traded factors. Out-of-sample test assets are the 26 long-short factors detailed in Appendix IA1.4.2 as well as 17 long portfolios based on FF17 industry sorts. Portfolio returns are calculated by weighting call options by the underlyings' capped market capitalization. RMSE and MAPE are based on returns standardized to an annual volatility of 100%.

Panel A: In-Sample Pricing, Test Assets: 51 Factors and 25 IVRV-DOI portfolios									
BMA-SDF	RMSE	MAPE	R^2_{ols}	R^2_{gls}	Model	RMSE	MAPE	R^2_{ols}	R^2_{gls}
10%-SR_{pr}	1.005	0.77	0.054	0.014	51 factors	0.046	0.027	0.998	0.968
25%-SR_{pr}	0.894	0.68	0.231	0.085	CAPM	1.034	0.801	-0.004	0.002
40%-SR_{pr}	0.771	0.6	0.425	0.177	HVX	0.734	0.554	0.489	0.211
60%-SR_{pr}	0.611	0.488	0.639	0.298	ZHCT	0.91	0.713	0.263	0.058
75%-SR_{pr}	0.508	0.414	0.75	0.375	AN	0.901	0.692	0.229	0.077
90%-SR_{pr}	0.431	0.358	0.82	0.435	TW	0.857	0.683	0.428	0.333

Panel B: Out-of-Sample Pricing, Test Assets: 26 Factors and 17 Industry Portfolios									
BMA-SDF	RMSE	MAPE	R^2_{ols}	R^2_{gls}	Model	RMSE	MAPE	R^2_{ols}	R^2_{gls}
10%-SR_{pr}	1.017	0.661	0.071	-0.027	51 factors	1.524	1.17	-1.086	-7.292
25%-SR_{pr}	0.908	0.606	0.26	0.08	CAPM	1.042	0.687	0.026	-0.018
40%-SR_{pr}	0.806	0.548	0.417	0.177	HVX	0.82	0.674	0.396	0.317
60%-SR_{pr}	0.691	0.471	0.571	0.286	ZHCT	0.794	0.633	0.434	0.065
75%-SR_{pr}	0.624	0.421	0.65	0.347	AN	0.964	0.677	0.165	0.055
90%-SR_{pr}	0.576	0.386	0.702	0.389	TW	0.926	0.694	0.23	0.101

Table IA6.4: Cross-sectional pricing performance - Puts, market capitalization weighting.

This table reports four performance measures of cross-sectional pricing for different factor models. For the BMA-SDF, prices of risk are estimated using the methodology outlined in Section 2. For the benchmark models, we use GMM with a GLS weighting matrix to estimate risk prices. Benchmark models are described in Appendix IA2. *CAPM* refers to a one-factor model utilizing only `ew_ret`, whereas *51 factors* utilizes all 30 traded and 21 non-traded factors. Out-of-sample test assets are the 26 long-short factors detailed in Appendix IA1.4.2 as well as 17 long portfolios based on FF17 industry sorts. Portfolio returns are calculated by weighting put options by their underlyings' capped market capitalization. RMSE and MAPE are based on returns standardized to an annual volatility of 100%.

Panel A: In-Sample Pricing, Test Assets: 51 Factors and 25 IVRV-DOI portfolios									
BMA-SDF	RMSE	MAPE	R^2_{ols}	R^2_{gls}	Model	RMSE	MAPE	R^2_{ols}	R^2_{gls}
10%-SR_{pr}	1.102	0.834	0.076	0.03	51 factors	0.086	0.046	0.994	0.91
25%-SR_{pr}	0.932	0.697	0.299	0.136	CAPM	1.124	0.878	-0.015	0.004
40%-SR_{pr}	0.762	0.574	0.523	0.269	HVX	0.92	0.705	0.319	0.234
60%-SR_{pr}	0.554	0.422	0.747	0.427	ZHCT	0.859	0.632	0.392	0.116
75%-SR_{pr}	0.424	0.332	0.851	0.518	AN	1.022	0.803	0.168	0.043
90%-SR_{pr}	0.325	0.266	0.912	0.59	TW	0.925	0.76	0.481	0.408
Panel B: Out-of-Sample Pricing, Test Assets: 26 Factors and 17 Industry Portfolios									
BMA-SDF	RMSE	MAPE	R^2_{ols}	R^2_{gls}	Model	RMSE	MAPE	R^2_{ols}	R^2_{gls}
10%-SR_{pr}	0.903	0.615	0.106	-0.213	51 factors	1.107	0.846	-0.344	-2.727
25%-SR_{pr}	0.765	0.55	0.357	0.006	CAPM	0.952	0.681	0.005	-0.116
40%-SR_{pr}	0.642	0.486	0.547	0.163	HVX	0.977	0.786	-0.048	0.138
60%-SR_{pr}	0.51	0.398	0.715	0.303	ZHCT	0.626	0.528	0.57	0.11
75%-SR_{pr}	0.436	0.347	0.792	0.373	AN	0.91	0.671	0.091	-0.141
90%-SR_{pr}	0.386	0.313	0.837	0.419	TW	0.83	0.63	0.244	-0.064

IA7 Margin-adjusted returns

Definition of margin-adjusted returns

In the main analyses, we scale the delta-hedged portfolio gain by the cash requirement to enter into a delta-hedged option position. In the following, we change the return definition to incorporate margin requirements following Bali et al. (2023). Precisely, we change equation (6) to

$$r_{t,t+\tau} = \frac{\Pi(t, t+\tau)}{M_t},$$

where $M_t > 0$ denotes the margin requirement for sustaining a delta-hedged option position from t to $t + \tau$. For the exact margin requirements, we adopt the CBOE minimum margin for customer accounts.[3] Also assuming a 50% margin requirement for long and short positions in the underlying stock, the margin requirement is given as

$$M_t = \begin{cases} O_t + 0.5|\Delta_t|S_t, & \text{for hedged long positions} \\ O_t + \max(0.1S_t, 0.2S_t - \max(0, K - S_t)) + 0.5|\Delta_t|S_t, & \text{for hedged short calls} \\ 0.1K + 0.5|\Delta_t|S_t, & \text{for hedged short puts} \end{cases}$$

where K denotes the strike price, O_t the option price, and S_t is the price of the underlying stock.

[3]See `https://www.cboe.com/us/options/strategy_based_margin`.

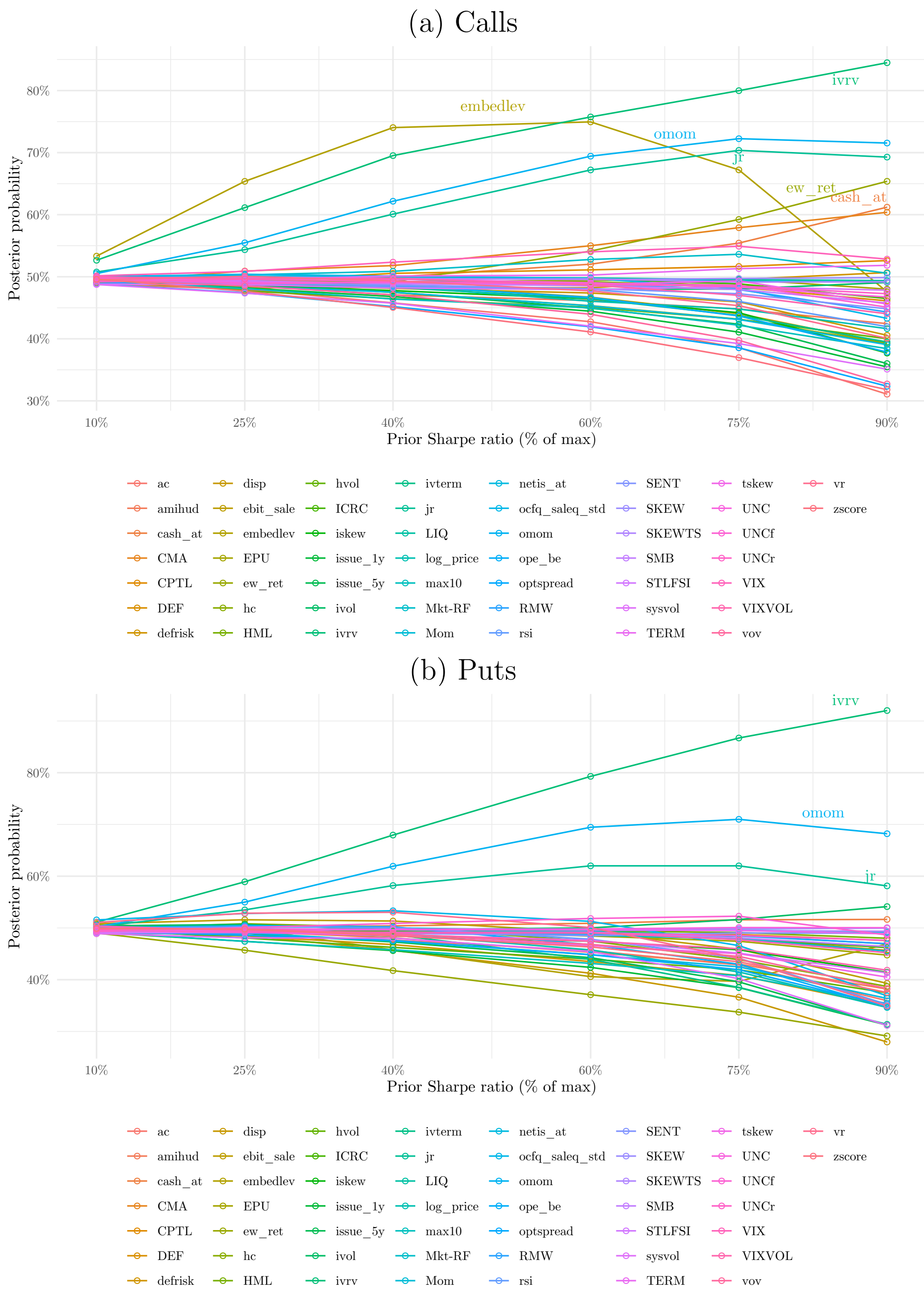


Fig. IA7.1. Posterior factor inclusion probabilities with margin-adjusted returns.

Notes: This figure shows posterior factor probabilities $\mathbb{E}[\gamma_j|\text{data}]$ estimated with the BMA approach outlined in Section 2. Test asset returns are calculated by equally-weighting margin-adjusted returns. Results for calls are shown in (a) and for puts in (b). All other specifications are detailed in Figure 1.

Table IA7.1: Cross-sectional pricing performance - Calls, margin-adjusted returns.

This table reports four performance measures of cross-sectional pricing for different factor models. For the BMA-SDF, prices of risk are estimated using the methodology outlined in Section 2. For the benchmark models, we use GMM with a GLS weighting matrix to estimate risk prices. Benchmark models are described in Appendix IA2. *CAPM* refers to a one-factor model utilizing only `ew_ret`, whereas *51 factors* utilizes all 30 traded and 21 non-traded factors. Out-of-sample test assets are the 26 long-short factors detailed in Appendix IA1.4.2 as well as 17 long portfolios based on FF17 industry sorts. Portfolio returns are calculated with equal call option weighting and margin-adjusted returns. RMSE and MAPE are based on returns standardized to an annual volatility of 100%.

Panel A: In-Sample Pricing, Test Assets: 51 Factors and 25 IVRV-DOI portfolios									
BMA-SDF	RMSE	MAPE	R^2_{ols}	R^2_{gls}	Model	RMSE	MAPE	R^2_{ols}	R^2_{gls}
10%-SR_{pr}	1.152	0.861	0.049	0.026	51 factors	0.046	0.023	0.998	0.98
25%-SR_{pr}	0.974	0.723	0.261	0.126	CAPM	1.242	0.968	0.067	0.002
40%-SR_{pr}	0.799	0.592	0.493	0.246	HVX	0.999	0.75	0.425	0.237
60%-SR_{pr}	0.613	0.462	0.705	0.391	ZHCT	1.074	0.786	0.095	0.033
75%-SR_{pr}	0.494	0.378	0.809	0.493	AN	1.183	1.027	0.436	0.12
90%-SR_{pr}	0.367	0.283	0.894	0.604	TW	1.529	1.326	-0.378	0.174

Panel B: Out-of-Sample Pricing, Test Assets: 26 Factors and 17 Industry Portfolios									
BMA-SDF	RMSE	MAPE	R^2_{ols}	R^2_{gls}	Model	RMSE	MAPE	R^2_{ols}	R^2_{gls}
10%-SR_{pr}	1.047	0.843	0.059	-0.058	51 factors	1.181	0.865	-0.196	-3.235
25%-SR_{pr}	0.922	0.727	0.271	0.088	CAPM	1.05	0.846	0.053	-0.167
40%-SR_{pr}	0.788	0.609	0.467	0.204	HVX	0.773	0.621	0.487	0.142
60%-SR_{pr}	0.663	0.505	0.623	0.3	ZHCT	0.93	0.721	0.257	0.034
75%-SR_{pr}	0.598	0.448	0.693	0.343	AN	0.81	0.608	0.437	-0.193
90%-SR_{pr}	0.544	0.398	0.746	0.371	TW	1.14	0.956	-0.116	0.1

Table IA7.2: Cross-sectional pricing performance - Puts, margin-adjusted returns.

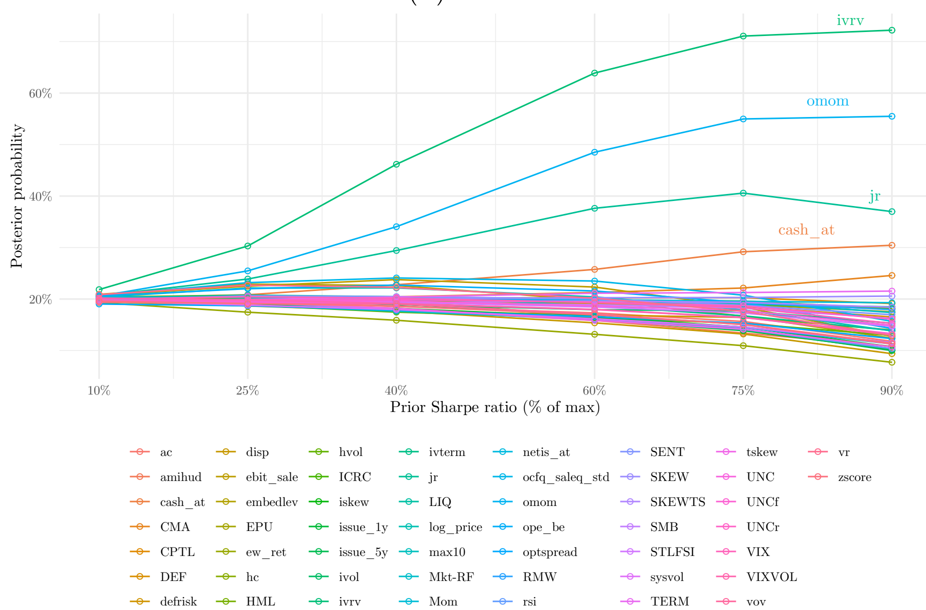
This table reports four performance measures of cross-sectional pricing for different factor models. For the BMA-SDF, prices of risk are estimated using the methodology outlined in Section 2. For the benchmark models, we use GMM with a GLS weighting matrix to estimate risk prices. Benchmark models are described in Appendix IA2. *CAPM* refers to a one-factor model utilizing only `ew_ret`, whereas *51 factors* utilizes all 30 traded and 21 non-traded factors. Out-of-sample test assets are the 26 long-short factors detailed in Appendix IA1.4.2 as well as 17 long portfolios based on FF17 industry sorts. Portfolio returns are calculated with equal put option weighting and margin-adjusted returns. RMSE and MAPE are based on returns standardized to an annual volatility of 100%.

Panel A: In-Sample Pricing, Test Assets: 51 Factors and 25 IVRV-DOI portfolios									
BMA-SDF	RMSE	MAPE	R^2_{ols}	R^2_{gls}	Model	RMSE	MAPE	R^2_{ols}	R^2_{gls}
10%-SR_{pr}	1.487	1.16	0.183	0.045	51 factors	0.032	0.017	1	0.99
25%-SR_{pr}	1.173	0.884	0.489	0.14	CAPM	1.54	1.231	0.293	0.016
40%-SR_{pr}	0.952	0.695	0.647	0.235	HVX	1.144	0.831	0.579	0.243
60%-SR_{pr}	0.715	0.525	0.789	0.355	ZHCT	1.1	0.785	0.51	0.106
75%-SR_{pr}	0.561	0.42	0.867	0.44	AN	1.509	1.144	0.217	0.079
90%-SR_{pr}	0.42	0.329	0.924	0.533	TW	1.211	1.032	0.331	0.341

Panel B: Out-of-Sample Pricing, Test Assets: 26 Factors and 17 Industry Portfolios									
BMA-SDF	RMSE	MAPE	R^2_{ols}	R^2_{gls}	Model	RMSE	MAPE	R^2_{ols}	R^2_{gls}
10%-SR_{pr}	1.154	0.973	0.213	-0.183	51 factors	2.439	1.759	-2.515	-12.93
25%-SR_{pr}	0.854	0.717	0.569	0.029	CAPM	1.086	0.919	0.303	-0.425
40%-SR_{pr}	0.688	0.574	0.721	0.19	HVX	0.915	0.723	0.506	-0.079
60%-SR_{pr}	0.546	0.445	0.824	0.338	ZHCT	0.694	0.546	0.715	0.073
75%-SR_{pr}	0.461	0.365	0.874	0.415	AN	1.197	1.012	0.154	-0.32
90%-SR_{pr}	0.395	0.304	0.908	0.482	TW	1.059	0.924	0.338	0.269

IA8 Conservative prior beliefs on factor inclusion

(a) Calls



(b) Puts

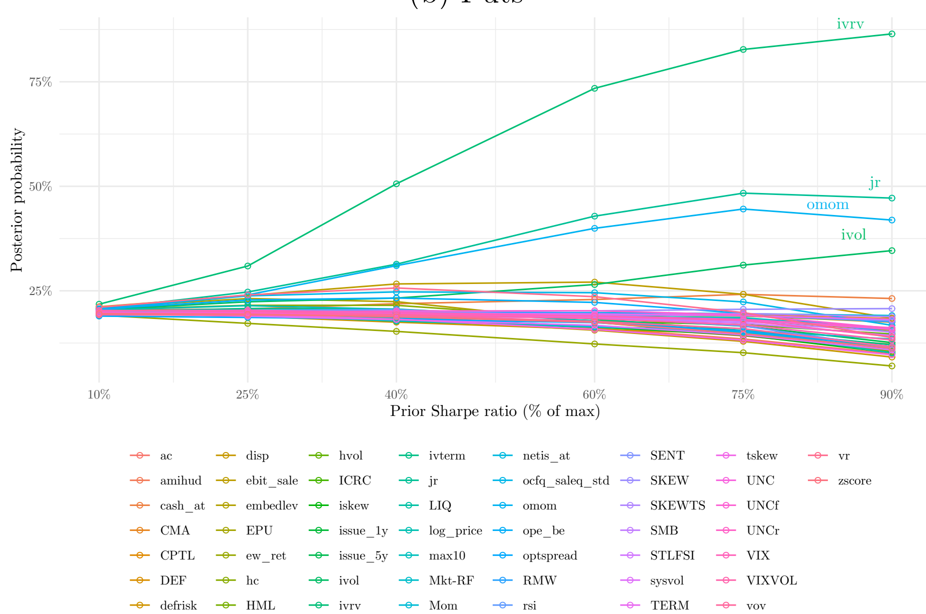


Fig. IA8.1. Posterior factor inclusion probabilities for more conservative prior beliefs on factor inclusion.

Notes: This figure shows posterior factor probabilities $\mathbb{E}[\gamma_j|\text{data}]$ estimated with the BMA approach outlined in Section 2. More conservative prior beliefs on factor inclusion are employed by initially drawing factor inclusion probabilities from a $Beta(3, 12)$ distribution. Results for calls are shown in (a) and for puts in (b). All other specifications are detailed in Figure 1.

Table IA8.1: Cross-sectional pricing performance - Calls, $Beta(3,12)$.

This table reports four performance measures of cross-sectional pricing for different factor models. For the BMA-SDF, prices of risk are estimated using the methodology outlined in Section 2. For the benchmark models, we use GMM with a GLS weighting matrix to estimate risk prices. Benchmark models are described in Appendix IA2. *CAPM* refers to a one-factor model utilizing only `ew_ret`, whereas *51 factors* utilizes all 30 traded and 21 non-traded factors. Out-of-sample test assets are the 26 long-short factors detailed in Appendix IA1.4.2 as well as 17 long portfolios based on FF17 industry sorts. Portfolio returns are calculated with equal call option weighting. Initial factor inclusion probabilities are drawn from a $Beta(3, 12)$ distribution. RMSE and MAPE are based on returns standardized to an annual volatility of 100%.

Panel A: In-Sample Pricing, Test Assets: 51 Factors and 25 IVRV-DOI portfolios									
BMA-SDF	RMSE	MAPE	R^2_{ols}	R^2_{gls}	Model	RMSE	MAPE	R^2_{ols}	R^2_{gls}
10%-SR_{pr}	1.407	1.076	0.053	0.041	51 factors	0.078	0.042	0.997	0.934
25%-SR_{pr}	1.238	0.946	0.234	0.101	CAPM	1.421	1.104	0.008	0.028
40%-SR_{pr}	1.057	0.814	0.433	0.189	HVX	0.971	0.686	0.536	0.265
60%-SR_{pr}	0.807	0.624	0.669	0.323	ZHCT	1.217	0.963	0.307	0.077
75%-SR_{pr}	0.643	0.498	0.79	0.408	AN	1.225	1.031	0.237	0.13
90%-SR_{pr}	0.528	0.418	0.858	0.471	TW	1.13	0.888	0.513	0.375

Panel B: Out-of-Sample Pricing, Test Assets: 26 Factors and 17 Industry Portfolios									
BMA-SDF	RMSE	MAPE	R^2_{ols}	R^2_{gls}	Model	RMSE	MAPE	R^2_{ols}	R^2_{gls}
10%-SR_{pr}	1.473	0.977	0.07	-0.029	51 factors	1.534	1.113	-0.007	-3.914
25%-SR_{pr}	1.286	0.866	0.291	0.074	CAPM	1.491	1.012	0.048	-0.011
40%-SR_{pr}	1.106	0.761	0.476	0.168	HVX	1.029	0.863	0.546	0.298
60%-SR_{pr}	0.908	0.625	0.647	0.278	ZHCT	1.078	0.825	0.502	0.044
75%-SR_{pr}	0.796	0.539	0.728	0.336	AN	1.249	1.007	0.332	0.057
90%-SR_{pr}	0.724	0.48	0.776	0.372	TW	1.155	0.877	0.429	0.009

Table IA8.2: Cross-sectional pricing performance - Puts, $Beta(3,12)$.

This table reports four performance measures of cross-sectional pricing for different factor models. For the BMA-SDF, prices of risk are estimated using the methodology outlined in Section 2. For the benchmark models, we use GMM with a GLS weighting matrix to estimate risk prices. Benchmark models are described in Appendix IA2. *CAPM* refers to a one-factor model utilizing only `ew_ret`, whereas *51 factors* utilizes all 30 traded and 21 non-traded factors. Out-of-sample test assets are the 26 long-short factors detailed in Appendix IA1.4.2 as well as 17 long portfolios based on FF17 industry sorts. Portfolio returns are calculated with equal put option weighting. Initial factor inclusion probabilities are drawn from a $Beta(3, 12)$ distribution. RMSE and MAPE are based on returns standardized to an annual volatility of 100%.

Panel A: In-Sample Pricing, Test Assets: 51 Factors and 25 IVRV-DOI portfolios									
BMA-SDF	RMSE	MAPE	R^2_{ols}	R^2_{gls}	Model	RMSE	MAPE	R^2_{ols}	R^2_{gls}
10%-SR_{pr}	1.591	1.239	0.053	0.025	51 factors	0.035	0.02	0.999	0.988
25%-SR_{pr}	1.38	1.066	0.24	0.098	CAPM	1.568	1.247	-0.008	0.011
40%-SR_{pr}	1.157	0.885	0.444	0.203	HVX	1.346	0.97	0.268	0.252
60%-SR_{pr}	0.871	0.657	0.68	0.356	ZHCT	1.229	0.919	0.362	0.108
75%-SR_{pr}	0.688	0.526	0.799	0.449	AN	1.508	1.153	0.104	0.072
90%-SR_{pr}	0.565	0.44	0.863	0.511	TW	1.36	1.081	0.444	0.376

Panel B: Out-of-Sample Pricing, Test Assets: 26 Factors and 17 Industry Portfolios									
BMA-SDF	RMSE	MAPE	R^2_{ols}	R^2_{gls}	Model	RMSE	MAPE	R^2_{ols}	R^2_{gls}
10%-SR_{pr}	1.396	0.958	0.078	-0.344	51 factors	1.532	1.192	-0.111	-3.616
25%-SR_{pr}	1.192	0.846	0.328	-0.125	CAPM	1.403	1	0.069	-0.138
40%-SR_{pr}	0.995	0.726	0.532	0.072	HVX	1.371	1.113	0.111	0.234
60%-SR_{pr}	0.786	0.587	0.708	0.271	ZHCT	0.885	0.733	0.629	0.118
75%-SR_{pr}	0.665	0.514	0.791	0.378	AN	1.41	1.012	0.059	-0.065
90%-SR_{pr}	0.588	0.465	0.837	0.449	TW	1.227	0.934	0.289	-0.111

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